Statistical Studies in the New York Money-Market

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STATISTICAL STUDIES

IN THE

NEW YORK MONEY-MARKET

PRECEDED BY A

BRIEF ANALYSIS UNDER THE THEORY OF

MONEY AND CREDIT

WITH

STATISTICAL TABLES, DIAGRAMS AND FOLDING CHART

BY

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PREFACE

In the following pages an attempt is made to apply the mathematical methods of interpolation and correlation to the financial statistics of discount rates and banking items as published weekly by the financial journals.

In studying correlation the methods of Professor Karl Pearson have been of very great use. Although Professor Pearson's writings have been largely in connection with the biological problems of evolution, his statistical methods have been found to apply satisfactorily to the problems presented under the theory of money and credit. I have tried as far as possible to subordinate the mathematical side of the work, and when for clearness it was necessary to introduce some mathematical formulas I have tried to state the logic of the formulas also in words and to express the meaning of the tables graphically by charts.

I am indebted to Professor Sumner and Professor Schwab for very material assistance; and to my mother for help in revising the proof and in verifying the statistical tables.

Yale University, New Haven,
April 27, 1901.

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PART I

BRIEF ANALYSIS UNDER THE THEORY OF MONEY AND CREDIT

NEW YORK MONEY-MARKET

PART I

Brief Analysis under the Theory of Money and Credit

§ 1. In any short time on a limited habitat, there are two elements in exchange, *commodities* and the *media of exchange*. Commodities consist of goods and labor. Media of exchange consist of money and credit.

With the two elements commodities and media of exchange there are four permutations,—(1) *Commodities against Commodities*, (2) *Commodities against Media of Exchange*, (3) *Media of Exchange against Commodities*, (4) *Media of Exchange against Media of Exchange*. In the end, men simply wish to exchange commodities.

Obviously this reduces to three cases:—*

Case I. Commodities against Commodities, i. e. *Barter*.

Case II. Commodities against the Media of Exchange, i. e. *Exchange*. The principal Media of Exchange are two, Money and Banking Devices.

Case III. Media of Exchange against Media of Exchange, i. e. *Money Changing and Banking*.

§ 2. If we mark off a limited area and limited time, we may throw the three cases into one equation representing the societary circulation. We may use letters in the following senses to represent the various factors.

C—commodities;

C_c—when bartered;

C_{md}—when exchanged against the media of exchange.

M_d—media of exchange.

P_{md}—prices reckoned in the media of exchange.

* Cf. Newcomb, *Principles*, Chapter XV.

P_c—prices reckoned in other commodities.

D—rate of discount on "futures" in the media of exchange (i. e. loans or "paper") bought by the banks.

M_{df}—"futures" in the media of exchange.*

M_{dp}—"spot" media of exchange.

M_{dc}—stock of money proper in *circulation.*

M_{db}—stock of money proper in the *total reserves* of the banks.

V_c—rate of turnover of money in circulation during the time (T).

V_b—rate of turnover of money in bank-vaults in indirectly effecting exchange.

We may now write the following equality as a first approximation:—†

$$\underset{\Sigma P_cC_c}{\text{barter}} + \underset{\Sigma P_{md}C_{md}}{\text{exchange}} - \underset{\Sigma(1-D)M_{df}}{\text{banking expenditures}} + \underset{\Sigma M_{dp}}{\text{receipts}} = \underset{\Sigma P_cC_c}{\text{barter}} + \underset{\Sigma M_{dc}V_cT}{\text{money paid in exchange}} + \underset{\Sigma M_{db}V_bT.}{\text{banking devices paid in exchange}}$$

Inasmuch as the part of business transactions effected by barter is relatively small, we may, to simplify the work, neglect both of the ΣP_cC_c terms. We then have the following equation:—

$$\Sigma P_{md}C_{md} - \Sigma(1-D)M_{df} + \Sigma M_{dp} = \Sigma M_{dc}V_cT + \Sigma M_{db}V_bT.$$

This is the general equation of the societary circulation developed by Professor Newcomb and other writers in a slightly different form. President Hadley,‡ in treating of the monetary aspect of this subject, has already brought into common use in his text-book the formula,

$$R \times M = P \times T,$$

where "the amount of money in the country" is "repre-

* Cf. Schwab, *Unpublished Lectures.*

† Cf. Fisher, *The Rôle of Capital in Economic Theory*, Economic Journal, December, 1897. "Gifts, bequests, charity, taxes, etc.," as well as failure of banking expenditures to exactly equal receipts during a short time makes this statement a rough approximation.

‡ *Economics*, p. 196.

sented by M and its rapidity of circulation by R." The price level P is a percentage of the price level of a preceding year "treated as unity." Transactions of the given year, "estimated at the prices of the 'standard year,' are represented by T."

§ 3. In the United States the basis of the media of exchange is gold coin, as now defined by the New Currency Act of March 1900. At a given time, there is a certain amount of gold coin in the country which does not fluctuate very widely from day to day. It may, of course, during longer periods change in amount owing to exports, loss, use by arts, imports, production and coinage.

Increase by (1) Imports;	Decrease by (1) Exports;
(2) Coinage.	(2) Loss;
	(3) Use by Arts.

The stock of gold may be said to be located in three places in the country, in the

(1) U. S. Treasury;
(2) Banks;
(3) Elsewhere.

All this is perfectly obvious and is stated simply to call attention to facts which later will be used.

With this apology, let us look at a sample U. S. Treasury statement* showing the amount of money in the United States on July 1, 1899.

Table No. 1.	General stock coined or issued, in millions.	Proportionate part.
Gold coin, including bullion in Treasury, . . .	$963	35%
Standard silver dollars, including bullion in Treasury,	564	20
Subsidiary silver, ...	75	3
Gold certificates, ...	34	1
Silver certificates, ...	406	15
Treasury notes, act July 14, 1890, ...	94	3
United States notes, ...	347	12
Currency certificates, act June 8, 1872, ...	21	1
National-bank notes, ...	241	9
Total, ...	$2,745	100%

* Report of the Secretary of the Treasury for 1899, Table M.

According to this statement, gold constitutes 35% of the stock of money, silver coin and bullion 20%, silver certificates 15%, United States notes 12%, bank notes 9%, and other forms 9%.

The distribution of this fund between the U. S. Treasury, the total reserves of the national banks, reserves of all other non-national banks and circulation among the people is roughly gauged by Table No. 2.

Table No. 2.	In millions.	Proportionate part.
In Treasury (Sept. 1, 1899), ...	$931	33%
National Banks (Sept. 7, 1899), ...	466	16
All other banks, ...	210	8
Circulation, ...	1,228	43
Total, ...	$2,835	100%

All these figures are in error owing to the methods of conjecture employed by the U. S. Treasury officials, but as we study further we shall find that it is not total amounts we care for, but changes on the margin. The really important problem to solve is the relation existing between prices, volume of transactions and media of exchange. To predict prices is the ultimately useful goal. Increasing transactions, it is said, make more exchange-work for the standard money and its dependent instruments of credit, enhancing in some degree the value of money, and consequently causing prices to fall. All this is very general and lacks the quantitative test.

§ 4. Without knowing the correct figures, we may, nevertheless, write out the equation of the relations. In this section, let us designate the various factors by the following letters:—

Media of Exchange.	Letter.	In Treasury.	In Banks. National.	In Banks. Non-National.	In Circulation.
Gold, ...	G	G_t	G_b	G_p	G_c
Silver, ...	S	S_t	S_b	S_p	S_c
Silver certificates, . .	C	C_t	C_b	C_p	C_c
United States notes, .	N	N_t	N_b	N_p	N_c
Bank notes, . . .	B	B_t	B_b	B_p	B_c
Other forms, . . .	L	L_t	L_b	L_p	L_c

The amount of money piled up in the Treasury, which can do no direct exchange-work as long as it lies in the vaults, is represented by the following expression:—

$$(G_t+S_t+C_t+N_t+B_t+L_t).$$

The gold, however, may be said to do *indirectly* the exchange-work of all fiduciary money and instruments dependent on it.

In the vaults of the national banks, we may represent the same as follows:—

$$(G_b+S_b+C_b+N_b+B_b+L_b).$$

The holdings of currency by all other banks may be tokened by use of the subscript (p):—

$$(G_p+S_p+C_p+N_p+B_p+L_p).$$

Elsewhere circulating among the people, we have the remaining amount:—

$$(G_c+S_c+C_c+N_c+B_c+L_c).$$

The sum of these four expressions is, of course, the total amount of currency in the country.

§ 5. So far we have been dealing with quantities of currency in existence at a time. The next step is to throw into exact form the theoretical exchange-work done by the media of exchange, as represented by the above expressions. The *exchange-work* accomplished during a time is found by multiplying the quantities of different kinds of currency by their respective "numbers of times of turnover" during the given time.

I. *Money in Circulation.* During the day, the various kinds of money pass from hand to hand in exchange for goods at different velocities. By velocity we mean the number of times the dollar exchanges for goods per unit of time. The exchange-work done for a short time, during which we may consider the sum of money in circulation relatively constant, is given by the following expression:—

$$T(G_cV_g+S_cV_s+C_cV_c+N_cV_n+B_cV_b+L_cV_l),$$

where (V) represents the average velocity* of the kind of money whose subscript it bears, and (T) the time.

It is probable that the government will, in course of time, institute investigations to determine the velocities of different kinds of money within limits. Such investigations could hardly fail to have extremely fruitful results. For much misunderstanding in monetary discussions arises in exactly this lack of *statistical data.*

It follows that:—

$$\frac{T(G_c V_g + S_c V_s + C_c V_c + N_c V_n + B_c V_b + L_c V_l)}{T(G_c + S_c + C_c + N_c + B_c + L_c)} = V,$$

where V denotes the weighted average velocity of the whole stock of money in circulation for the time (T).

We may now write the exchange-work accomplished by money in passing from hand to hand thus:—

$$E_c = TV(G_c + S_c + C_c + N_c + B_c + L_c).$$

II. *Money in bank vaults.* The next step is to develop a like equation for the exchange-work of money in banks.†

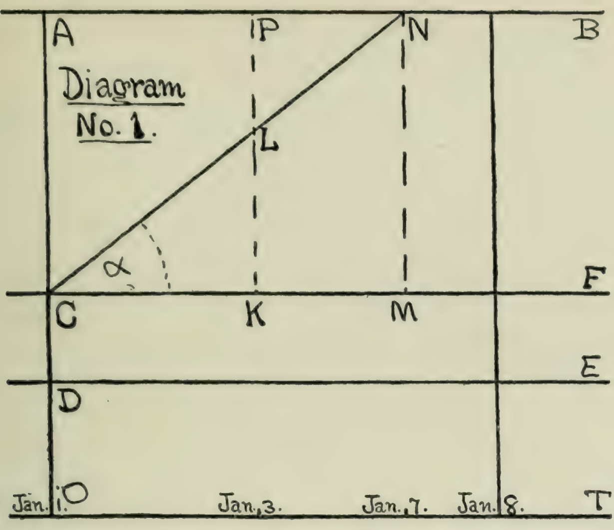
On the books of the banks are all the records of all transactions effected by checks and drafts. That a check or draft be honored, a deposit must stand on the books of the bank subject to the order of the drawer. As the days pass, the deposits of yesterday are checked out, and new deposits are entered. The weekly statements of the New York Associated Banks furnish a concrete example.

In the figure below, let OT be the time line, OD the average specie holdings for the week January 1 to January 8, DC the average legal tender holdings. It follows that OD+DC, i. e. OC, equals the total reserves. AC represents the average deposits.

As the days pass, old deposits are checked out, and new deposits are entered. There were AC deposits on January 1. On January 3, there were PL deposits of the old AC

* Cf. Fisher, *The Rôle of Capital in Economic Theory*, Economic Journal, 1897, pp. 520.

† Cf. § 2.



deposits left, and LK represents new deposits. On January 7, all the old deposits are cancelled, and NM new deposits stand on the books.

T = CM = time of turnover of the deposits AC.

The reciprocal $1/T = U$, the rate of turnover of each dollar of the deposits. It follows that

$T \tan a = CM \times NM/CM = T \times D/T =$ exchange-work done by the deposits AC.

In general, if D be the deposits at a time, U the rate of turnover,* and T a short time during which the deposits and rate of turnover vary little in amount, the exchange-work done is given by the expression

$$T \times D \times U.$$

The equation of societary circulation is generally given as

$$E = T(M \times V + D \times U), \dagger$$

where M represents money, V velocity of money, D deposits and U rate of turnover of deposits. This expression did not

* Cf. Pierre Des Essars, *La Vitesse de la Circulation de la Monnaie*, Jour. Soc. Statistique, Paris, April, 1895. Relation of V to *crises* is developed.

† Cf. Newcomb, *Principles;* Fisher, *Unpublished Lectures;* Gaines, *Unpublished Lectures.*

seem to me to readily lend itself to statistical investigation. In fact, interest attaches to the relation existing between deposits and the total reserves of the banks.

The Act of January 1875 prescribes that banks shall hold a minimum reserve against deposits. In the reserve cities, the reserves must be 25% of the deposits; for all other banks 15%. A bank in a reserve city may deposit 1/2 of its required reserve in any central reserve city bank, and any bank outside of the reserve cities may deposit 3/5 of its required reserves with other banks in the central reserve cities.

Consequently, with a given amount of total reserves, we may write the equations by which the maximum deposits with those reserves may be reached.

By R designate total reserves. The reserves of the country banks will be R_1, of the reserve city banks R_2, and of the central reserve city banks R_3, deposits D_1, D_2, D_3.

For COUNTRY BANKS, deposits are forbidden to exceed the ratio 100/15 to reserves. It follows that

$$\text{Maximum } D_1 = 100/15\ R_1.$$

But 3/5 of R_1 may consist of deposits in the central reserve cities. Consequently

$$\text{Maximum } D_1 = 100/15 \times 5/2\ R_{m_1} = 16.667\ R_{m_1}.$$

R_{m_1} stands for the lawful money reserve.

For the RESERVE CITY BANKS, the ratio is 100/25. Thus

$$\text{Maximum } D_2 = 100/25\ R_2.$$

Here 1/2 of R_2 may be deposited with the central reserve banks. Therefore

$$\text{Maximum } D_2 = 100/25 \times 2/1\ R_{m_2} = 8\ R_{m_2}.$$

In the same way for the CENTRAL RESERVE BANKS,

$$\text{Maximum } D_3 = 100/25 = 4\ R_{m_3}.$$

By adding together

$$\text{Max. } D_1 + \text{Max. } D_2 + \text{Max. } D_3,$$

we obtain the maximum lawful deposits attainable by the National Banking System.

Substituting the above values we have

$$\text{Maximum } D = 16.667\ R_{m_1} + 8\ R_{m_2} + 4\ R_{m_3}.$$

This then is the maximum towards which the deposits of the National Banking System are all the time tending. The nearer bank presidents can approximate this condition, the greater are their profits. If business presses hard on the maximum, discount rates rise. As business lags far behind the maximum, discount rates fall that the loan account may increase and so deposits increase.

If we take the Comptroller's return for October 2, 1890, we find by the equation:—

$$\begin{aligned}\text{Maximum } D &= 16.667\ R_{m_1} + 8\ R_{m_2} + 4\ R_{m_3}\\ &= \$1,172 + \$614 + \$369\\ &= \$2,154.8 \text{ millions.}\end{aligned}$$

Now the actual deposits were $1,758.7 millions, or 82% of the possible lawful maximum. It is never safe to approach too closely to the maximum, for the funds in country banks, reserve city banks and central reserve banks are all the time flowing from one to the other. At certain seasons, there is a strong ebb in one direction, and, at other seasons, opposite currents. (This will be shown later by statistical charts.) And so a margin must be left against these flows as well as against a possible increased demand of money for currency circulation.

To illustrate the variations, four recent returns of the Comptroller are reduced to the above percentage of the maximum.

December 1, 1898—94%	September 7, 1899—80%
February 4, 1899—72%	September 5, 1900—83%

These percentages measure the credit strain to which the banking system is at a time subjected. The lawgivers prescribed 100% as the safety limit for the stress.

If we let K_1, K_2, and K_3 stand for the percentage proportion the actual deposits D_1, D_2, and D_3 bear to the MAXIMUM D_1, MAXIMUM D_2 and MAXIMUM D_3, we may write for the whole country

$$\text{Maximum } D = D/K.$$

But we know

$$D = 16.667\ K_1R_{m_1} + 8\ K_2R_{m_2} + 4\ K_3R_{m_3}$$

as the more exact expression.

Further, let U stand for the weighted average rate of turnover of deposits for the whole country, and U_1, U_2, U_3, the velocities for country, reserve city, and central reserve city banks respectively. We may then write the familiar expression $D \times U$ in the following form:—

$$D \times U = 16.667\ K_1U_1R_{m_1} + 8\ K_2U_2R_{m_2} + 4\ K_3U_3R_{m_3}.$$

This gives the exchange-work equation. It stands for the exchange-work done per unit of time by the specie and legal tenders lying in the vaults of the national banks throughout the country.

III. The exchange-work equation for the remaining *State Banks*, *Private Banks* and *Trust Companies* may be worked out in a manner analogous to the above. If we let U_p represent the average weighted velocity of deposits, D_p deposits, K_p the percentage proportion as above between actual and maximum deposits, R_{mp} the total specie and legal tender holdings, and C_p the coefficient of maximum deposits to specie and legal tender holdings as fixed by the laws of the various states, the general equation of exchange-work may be written:—

$$D_pU_p = C_pK_pU_pR_{mp}.$$

We may now write the complete equation of exchange-work (E), done by money *directly* by passing from hand to hand against goods, and by money *indirectly* as the reserve in national and other banks. The letters are used in precisely the same senses as above. The equation comes by addition of the expressions already derived.

$$E = T < V_m(G_m + S_m + C_m + N_m + R_m + L_m) + 16.667\ K_1U_1R_{m_1} + 8\ K_2U_2R_{m_2} + 4\ K_3U_3R_{m_3} + C_pK_pU_pR_p >$$

This expression represents the media of exchange side, i. e. the right hand side of the general equation of the societary circulation developed in § 2.

§ 6. Let us consider for a moment the statistical data bearing upon these items.

The reports of the Treasurer of the United States contain an estimate of the amount of money of all kinds in circulation on the last day of each month. These estimates are conveniently collected in the Commercial Year-book of the Journal of Commerce.

K_1, K_2, K_3 are easily found from the Reports of the Comptroller of the Currency. R_{m_1}, R_{m_2}, R_{m_3} are likewise given in the same report.

V_m we do not know save for a limited set of observations.* We have indirect methods of judging its limitations, which, however, have been little worked.

C_p, K_p, and U_p are given in no one place. The Comptroller gives estimates of R_{mp}.

U_1, U_2, and U_3 we have no way of finding directly† at the present time. A limited range of statistics exists, collected by Dr. Gaines for some New York banks. There is also a series compiled for the Bank of France.‡ As a substitute, Dr. Gaines has suggested the ratio of clearings to average deposits. The Commercial and Financial Chronicle publishes weekly the clearings for the whole country. These ratios for the three classes of national banks we will designate by W_1, W_2, W_3.

$$W_1 = C_1/D_1, \quad W_2 = C_2/D_2, \quad W_3 = C_3/D_3.$$

* "A hundred such returns among students at Yale University indicated an average velocity of forty-five times a year, making the average length of time a dollar rests in one man's hands about eight days." (Fisher, cited above, pp. 520.)

† There is no more deserving a question for investigation by the Government in the whole subject of money and banking than the determination of this data with the aid of the banks. The question is not divorced from practical affairs; for it is so intimately connected with crises and price levels that *exact* knowledge will render safeguards possible.

‡ Pierre des Essars also gives annual averages for the Banks of France, Germany, Belgium, Portugal, Spain, Italy and Greece.

Five times a year, the national banks make returns of D_1, D_2, and D_3. In the case of D_3, however, the central reserve banks publish in addition weekly averages. This determines W_3 weekly. The ratio W_3 is smaller than U_3 by the amount of checks that cancel each other in the banks and fail to pass through the Clearing-house. It is probably unsafe to say that W_3 and U_3 vary together in fixed proportion, but there is no doubt that they vary in the same directions.

§ 7. We have now a complete statement of the monetary factors of the equation of the societary circulation. In a laboratory of economic statistics, we may imagine on file records of each one of these items for a long series of years. We might then write the exchange-work equation across the top of a wide sheet of paper and in columns below each letter fill in the appropriate value for each week. Such a statement would be but a systematic arrangement of facts. The interest of science demands more than an arrangement of facts. The predominating interest of all science is in discovering correlations, laws, which derived from past phenomena may be relied upon with some degree of probability to predict future phenomena.

Social utilities are served by relations found to exist between the motions of the heavenly bodies and the dimension time. The tides, time of sunset and sunrise, eclipses, all may be predicted for months and years. Time in economics is of no less interest in the business world.

In a search for correlations it is wise at the start to divide the work into four classes of problems:—

(1) Proportional relations at a time between the items of the equation showing the relative importance of each.

(2) Deviations in items at one time which are followed later by deviations in other items, thus showing anticipatory correlation.

(3) Changes in a single column coincident with the passage of a long series of years.

(4) Periodic changes in one column with the seasons of the year.

PART II

STATISTICAL STUDIES IN THE NEW YORK MONEY-MARKET

CHAPTER I

THE STATISTICS OF THE NEW YORK ASSOCIATED BANKS

§ 8. Theory and statistics are the two legs of economic science. Some theory in the past without statistics has notoriously gone lame. This lameness in the past had a justifiable palliation. There were no statistics to be had. To-day this lack of accurate statistics is rapidly vanishing. Trade journals, governments, states, cities, corporations, commissions—public, semi-public and private—are all throwing out daily and weekly masses of undigested statistics that are appalling.

In the past statisticians have too often been content to study such arrays of statistics by an average. Yearly averages and often averages for five and ten-year periods have been taken. The results of such conglomerations of many tendencies into a single index have been often small. An average is easy, often useful, but many times a senseless thing. In many cases, it must be confessed, the average tells very little. In the future, it will perhaps fall back to its place as only one of the several decisive quantities of the frequency curve of the array.

The great fault of a series of averages is the lack of continuity. This becomes glaring when quinquennial and biennial, or often annual, averages are used. In many investigations quarterly and monthly averages are not sufficiently continuous. The week is much nearer the ideal unit; for it combines practicability of handling with an approximate continuity. Yet in problems of correlating variations in stock quotations and the call discount rate at the stock exchange in times of serious disturbance to credit, a twenty or thirty minute average is necessary for results.

Such misuse of indices has brought down that sarcasm of Bernard Moses,—"We have plenty of statisticians, but no

statistics." The reverse is perhaps nearer the truth. There is an abundance of excellent statistics, but a narrow dispersion of statistical method as it exists in other branches of science. To make light of our statistics is to cut huge lumps from beneath the ground-work of economic science.

§ 9. In discussing the problems arising in a contemplated perfect table of statistics covering many years and prepared as suggested in Part I, we divided the problems for convenience into four classes. Two of these classes of problems consisted in correlating changes in different items, (i) at the same time, (ii) at different times, the one giving *immediate* and the other *anticipatory correlation*. The other two problems were concerned with changes in one item coincident (i) with the passage of a long lapse of time, and (ii) with recurrent or periodic time.

Unfortunately no such perfect table exists. The investigator must needs attempt the compiling of a part. The complete expression for the exchange-work (E), it will be remembered, was

$$[T < V_m(G_m + S_m + C_m + N_m + R_m + L_m) + 16.667\ K_1U_1R_{m_1} + 8\ K_2U_2R_{m_2} + K_3U_3R_{m_3} + C_pK_pU_pR_p >].$$

In the following investigation the factor R_{m_3} was split up. R_{m_3} stands for the total reserves of the central reserve city banks. It consists of the reserves of the Associated Banks of New York, Chicago and St. Louis. Inasmuch as the returns for New York are by far the most important, constituting the largest portion of the factor R_{m_3}, it seemed best to make a beginning with that series. It will be convenient to consider first the single item and later the problems of correlation.

§ 10. The *weekly statements** of the New York Associated Banks go back for over forty years. The method of preparing the statement is the so-called "system of averages." The

* "Weekly bank statements were not made by any banks until Aug. 6, 1853, when the New York banks began the custom, and others gradually followed." Sumner, *History of American Currency*, pp. 175–6.

typical bank which is a member of the Clearing House Association, prepares a table showing for each day of the week the amount of loans, deposits, specie and legal tenders held within its vaults. On the last day of the week, the bank adds up the six values for each item and divides the totals by six. If a holiday intervenes, the divisor is of course five.

The averages of each bank for the several items are reported to the Clearing House and there the weekly statements of the New York Associated Banks are compiled. Thus the items in the statement for loans or deposits are the sums of the amounts of loans or deposits reported by the several banks. The statement as issued by the Clearing House gives in addition the changes, positive or negative, with respect to the preceding week in each item.

The bank statement is often the "feature" in the Saturday market and, when it appears at about 11.30 A. M., frequently becomes the cause of an advance or of a decline in quotations, as the statement seems favorable or unfavorable to the operators. The theory is that smaller reserves (i. e. shrinkage in the sum of specie and legal tenders) lower the ratio of reserves to deposits, which by law shall not sink below 25%. The bankers' method of guarding against a too rapidly shrinking tendency in the total reserves is to raise discount rates. Higher discount rates tend necessarily to make the burden of carrying stocks on a margin greater and consequently "make" for lower prices. Inasmuch as, on the whole, increasing loans tend to increase deposits, all the items affect directly or indirectly the ratio.

The ways in which these items are correlated together and with the discount rates on call loans will appear later. It is enough here to note:—

(1) That the statistics are well prepared and represent the banking conditions at one of the most important financial centers of the world;

(2) That the weekly average affords very satisfactory continuity;

TABLE NO. 3.—WEEKLY AVERAGE TOTAL RESERVES OF THE NEW YORK ASSOCIATED BANKS IN MILLIONS.

Numerical Week.	Corresponding Month and Day.	1879	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
		− 3	− 4 From March on − 5	+ 1	0	− 1	− 2 From March on − 3	− 4	− 5	+ 1	0 From March on − 1	− 2	− 3	− 4	− 5 From March on − 6	0	− 1	− 2	− 3 From March on − 4	− 5	+ 1	0	− 1
1	Jan. 7	63	61	76	78	80	91	125	119	107	104	110	104	105	134	123	213	174	143	166	194	230	199
2	14	64	65	81	84	85	99	135	124	112	109	117	110	111	138	131	225	181	150	181	199	237	204
3	21	67	69	83	88	86	104	139	130	114	116	122	113	119	146	142	238	186	155	191	210	247	216
4	28	71	69	83	88	87	107	140	132	118	118	126	121	125	157	145	246	186	160	198	216	255	224
5	Feb. 4	73	69	84	85	85	108	142	134	118	119	127	122	127	163	142	250	173	162	200	218	257	230
6	11	69	69	83	82	84	111	143	133	117	117	123	118	124	162	140	219	167	163	197	217	258	231
7	18	66	71	81	77	81	111	141	133	113	114	127	115	123	164	135	207	167	160	194	210	259	229
8	25	63	75	73	73	79	110	138	130	108	110	126	111	120	164	128	208	162	151	199	205	258	226
9	Mar. 4	59	72	68	70	74	109	137	123	105	107	122	107	117	160	122	209	160	147	201	197	253	221
10	11	57	70	68	72	69	101	136	121	104	106	118	103	114	155	116	209	155	145	200	197	250	213
11	18	56	70	72	75	66	94	136	119	103	104	119	104	112	149	116	212	147	144	197	200	244	206
12	25	56	67	71	75	64	96	136	115	100	103	117	106	113	150	119	215	141	141	192	205	244	206
13	April 1	53	66	70	73	66	94	137	108	97	102	115	107	112	151	120	220	139	139	192	206	240	212
14	8	50	64	73	72	67	90	135	105	98	101	109	104	110	150	119	219	139	137	190	202	239	214
15	15	55	64	76	78	71	90	137	109	99	104	115	103	110	148	121	222	141	139	188	203	242	220
16	22	59	64	81	82	72	88	140	109	99	108	122	103	108	149	125	224	147	139	189	209	245	226
17	29	63	64	85	85	73	86	142	107	100	110	124	105	109	153	120	227	154	143	193	209	246	230
18	May 6	68	66	89	85	76	84	145	105	99	114	120	105	109	154	121	227	159	147	193	209	244	233
19	13	72	71	94	83	81	87	146	104	98	119	119	103	106	148	127	225	165	144	187	214	252	232
20	20	68	75	98	81	82	82	149	105	98	125	125	104	104	148	124	224	176	143	188	221	260	234
21	27	62	78	98	79	87	68	151	105	99	127	126	105	103	153	135	221	180	146	189	228	266	238
22	June 3	60	82	94	79	87	70	151	105	97	123	125	106	104	159	129	221	183	147	190	230	256	242
23	10	62	83	95	80	88	72	151	103	97	126	121	106	103	159	119	220	181	143	191	235	264	242
24	17	64	85	94	85	88	77	154	107	98	130	122	108	106	159	110	219	180	145	194	242	257	241
25	24	63	86	95	85	90	80	155	109	96	129	120	107	115	159	105	220	180	144	196	247	253	238
26	July 1	67	87	94	82	90	85	158	108	95	129	118	108	119	154	101	218	177	146	199	249	240	239

TABLE NO. 3 (CONTINUED).—WEEKLY AVERAGE TOTAL RESERVES OF THE NEW YORK ASSOCIATED BANKS IN MILLIONS.

Numerical Week.	Corresponding Month and Day.	1879	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
		− 3	− 4 From March on − 5	+ 1	0	− 1	− 2 From March on − 3	− 4	− 5	+ 1	0 From March on − 1	− 2	− 3	− 4	− 5 From March on − 6	0	− 1	− 2	− 3 From March on − 4	− 5	+ 1	0	− 1
27	July 8	69	87	94	88	87	89	157	105	99	128	116	107	117	152	95	219	175	145	193	243	231	236
28	15	71	90	99	91	90	99	160	108	100	132	118	110	115	148	94	221	175	148	194	239	232	241
29	22	71	92	98	89	91	105	161	109	100	130	118	109	121	151	96	221	181	148	199	228	231	245
30	29	74	89	98	85	90	107	161	109	99	130	116	108	121	154	91	218	185	142	203	227	226	250
31	Aug. 5	77	89	93	83	90	106	160	107	97	130	117	113	121	156	79	214	184	139	201	229	221	253
32	12	70	86	88	84	91	108	158	103	93	128	115	103	119	151	76	212	182	133	198	223	227	252
33	19	63	85	83	82	88	109	157	99	91	126	111	99	119	147	80	214	185	126	197	220	228	244
34	26	61	82	78	80	85	108	156	96	91	124	108	95	115	143	86	213	181	124	199	211	227	249
35	Sept. 2	61	81	78	74	85	107	155	94	91	120	110	96	114	139	92	212	183	122	199	203	222	253
36	9	60	80	80	74	83	106	150	93	90	114	114	96	110	135	96	208	179	120	195	190	211	253
37	16	59	80	80	74	83	104	147	94	90	114	111	92	110	130	105	207	170	120	187	182	205	248
38	23	62	80	80	72	81	105	144	94	91	113	107	99	109	128	114	207	162	122	179	183	203	241
39	30	60	78	74	70	79	104	141	96	95	117	105	116	105	125	122	207	160	126	172	191	198	234
40	Oct. 7	58	78	71	72	80	107	138	93	96	115	101	115	104	124	129	207	151	130	170	196	196	226
41	14	59	80	70	74	81	111	137	92	96	114	101	105	108	120	138	208	147	130	168	201	195	220
42	21	59	80	74	74	78	110	133	92	99	122	102	100	112	118	148	211	148	125	169	210	193	215
43	28	58	79	77	72	77	111	127	93	101	120	101	100	116	118	157	212	149	127	177	216	193	217
44	Nov. 4	58	79	76	72	77	110	125	94	99	118	102	100	117	118	164	212	150	129	181	211	189	216
45	11	57	79	76	68	79	113	121	93	98	115	100	95	111	117	162	210	151	124	179	209	183	212
46	18	65	77	75	70	82	119	121	96	96	115	101	96	114	116	182	212	152	137	181	213	184	218
47	25	69	76	73	67	84	122	123	99	95	115	102	95	119	118	190	214	151	148	184	214	191	225
48	Dec. 2	69	72	71	71	85	124	122	98	94	112	102	95	122	120	198	197	149	154	187	216	196	227
49	9	69	67	71	76	84	125	121	96	94	108	100	92	122	120	200	175	151	158	189	216	193	221
50	16	67	66	71	78	85	125	123	94	96	110	101	95	124	118	200	175	149	161	188	219	192	217
51	23	63	69	73	78	87	124	121	92	97	107	103	100	131	118	203	172	146	164	183	223	195	219
52	30	61	70 71	74	76	87	125	119	95 102	98	106	102	103	133	117 118	207	173	141	166	179 185	230	196	225

TABLE NO. 4.—WEEKLY AVERAGE LOANS OF THE NEW YORK ASSOCIATED BANKS IN MILLIONS.

Numerical Week.	Corresponding Month and Day.	1879	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
		− 3	− 4 From March on − 5	+ 1	0	− 1	− 2 From March on − 3	− 4	− 5	+ 1	0 From March on − 1	− 2	− 3	− 4	− 5 From March on − 6	0	− 1	− 2	− 3 From March on − 4	− 5	+ 1	0	− 1
1	Jan. 7	234	277	304	319	317	331	298	340	348	360	392	400	386	439	441	419	493	466	491	610	714	678
2	14	231	276	303	320	318	330	296	340	348	356	386	400	384	444	440	418	490	458	491	611	717	676
3	21	233	277	308	321	317	333	295	339	351	355	389	403	383	446	447	420	490	454	491	623	720	681
4	28	234	280	311	323	317	334	294	339	353	356	393	400	386	447	445	419	490	448	490	626	726	688
5	Feb. 4	238	283	316	329	316	339	294	338	359	363	400	404	390	454	465	420	490	447	489	634	742	700
6	11	242	290	317	328	321	342	295	339	366	366	408	412	398	461	464	433	485	448	498	640	750	721
7	18	244	290	321	329	323	346	299	345	367	366	408	414	402	466	463	439	483	452	500	647	759	734
8	25	244	290	317	325	325	345	298	348	368	367	409	415	404	477	459	441	483	458	499	642	772	746
9	Mar. 4	247	294	298	321	327	344	299	350	369	368	413	420	404	481	453	439	484	463	498	628	781	755
10	11	248	297	296	314	325	348	302	355	370	369	417	406	405	489	445	440	489	464	502	618	778	763
11	18	246	297	300	312	320	351	304	359	369	370	420	405	404	493	439	443	489	467	506	607	774	752
12	25	244	294	301	311	313	348	301	360	365	369	421	404	408	495	434	446	485	468	506	600	779	739
13	April 1	240	291	300	313	310	346	301	356	366	369	421	405	410	490	434	444	483	465	504	596	780	743
14	8	236	291	305	314	311	348	303	350	371	368	420	408	413	490	434	450	480	465	503	588	778	756
15	15	230	288	306	313	310	348	302	350	369	367	417	407	414	491	431	457	481	466	503	580	768	762
16	22	231	284	306	310	311	347	302	351	363	363	417	404	412	494	429	459	481	466	504	573	761	767
17	29	231	279	304	311	313	343	298	352	361	364	416	400	407	492	426	461	481	467	505	570	760	775
18	May 6	239	280	311	315	316	342	297	351	365	366	418	400	404	493	426	465	485	471	506	571	777	788
19	13	243	281	318	316	315	333	299	351	365	364	417	402	403	494	421	467	489	474	505	574	771	787
20	20	254	279	324	316	318	327	299	347	366	362	413	401	399	492	417	467	495	476	506	582	764	788
21	27	258	272	332	318	316	313	296	342	364	364	412	400	393	491	416	467	500	473	505	590	746	793
22	June 3	257	273	341	318	318	310	293	342	365	364	412	397	389	489	417	465	503	475	508	602	747	800
23	10	258	276	347	318	321	303	296	343	365	366	414	397	386	493	414	465	503	477	512	611	757	807
24	17	256	279	347	317	322	296	297	344	366	372	416	395	383	494	411	466	508	474	514	610	773	810
25	24	256	286	345	319	326	293	299	347	365	374	417	396	386	497	406	468	513	475	519	613	779	810
26	July 1	254	286	350	323	328	293	304	351	364	377	417	397	390	495	414	470	513	475	522	621	787	808

TABLE NO. 4 (CONTINUED).—WEEKLY AVERAGE LOANS OF THE NEW YORK ASSOCIATED BANKS IN MILLIONS.

Numerical Week.	Corresponding Month and Day.	1879	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
		− 3	− 4 From March on − 5	+ 1	0	− 1	− 2 From March on − 3	− 4	− 5	+ 1	0 From March on − 1	− 2	− 3	− 4	− 5 From March on − 6	0	− 1	− 2	− 3 From March on − 4	− 5	+ 1	0	− 1
27	July 8	257	292	353	327	329	293	307	356	360	380	423	405	394	494	419	484	514	476	533	634	794	803
28	15	263	293	349	325	330	290	307	355	358	379	421	403	392	492	413	483	511	477	534	640	777	801
29	22	263	292	349	330	328	289	309	354	355	380	419	402	392	482	409	483	507	480	534	635	768	798
30	29	261	295	349	333	327	290	308	354	355	379	417	400	391	480	406	482	506	474	540	637	760	801
31	Aug. 5	267	298	351	335	327	288	306	354	356	382	413	402	390	485	408	482	509	470	543	649	753	804
32	12	273	305	351	337	327	289	313	358	353	386	415	406	391	489	411	485	511	468	550	659	747	808
33	19	274	306	350	338	329	288	315	355	351	388	416	402	394	490	407	486	511	465	555	667	748	815
34	26	264	311	343	336	328	288	318	348	348	389	412	398	395	492	404	489	514	459	559	672	757	817
35	Sept. 2	258	311	337	332	326	288	320	342	345	392	406	393	397	491	400	490	513	456	561	672	754	817
36	9	257	312	334	330	328	289	325	339	347	393	407	395	401	487	397	492	518	453	569	665	748	819
37	16	257	314	334	327	329	291	327	338	347	392	410	393	402	482	393	495	523	453	577	653	740	826
38	23	259	314	333	326	332	291	328	337	346	391	410	393	405	475	392	498	517	452	579	642	722	823
39	30	261	310	330	319	330	291	329	337	345	391	409	394	408	467	392	498	511	451	577	635	714	817
40	Oct. 7	266	309	326	315	326	291	331	340	348	396	407	402	406	465	393	500	510	453	572	636	711	817
41	14	269	314	318	312	328	291	332	344	352	397	403	407	403	463	395	500	507	456	572	646	706	808
42	21	268	316	311	310	328	292	335	344	351	394	398	406	405	460	394	501	504	456	569	657	701	798
43	28	269	317	309	312	325	292	340	342	350	394	395	402	406	452	397	500	502	450	562	667	696	793
44	Nov. 4	271	318	313	318	324	292	344	341	352	394	396	400	408	449	402	501	501	446	567	679	696	792
45	11	270	324	313	315	322	290	341	341	352	394	398	399	411	449	403	500	496	442	574	688	688	786
46	18	269	325	315	314	324	292	340	342	352	393	396	393	409	446	402	499	493	445	575	694	680	788
47	25	276	315	315	309	325	288	339	342	353	391	395	387	409	444	405	495	493	455	580	691	677	793
48	Dec. 2	273	314	315	305	326	286	341	347	354	391	396	385	412	443	409	499	490	464	594	698	682	805
49	9	273	306	315	304	328	288	339	351	353	389	394	386	418	445	412	508	490	472	598	702	681	806
50	16	276	294	313	307	327	290	339	352	350	386	390	386	418	443	415	507	493	484	608	709	676	793
51	23	278	293	313	310	328	294	338	349	350	388	393	386	423	442	416	498	490	487	607	712	673	787
52	30	278	292 298	315	311	328	296	337	343 344	357	389	395	385	429	438 438	418	493	478	488	611 608	718	674	797

(3) That the statistics are not eccentric and solely a satisfaction of a scientific curiosity; but that they hold and have held for over forty years a well recognized place of importance* in the financial world. These statements have been again and again for weeks at a time the absorbing "feature" of the security market. The "bull" market of 1899 and the panic of December 18, 1899 were, in the first case, attended by a series of most unusually favorable bank statements and, in the latter case, by a series of disastrous ones. Indeed, the importance of banking statistics is well known to the greater operators of the street, not only of our day, but it was so to operators of days long gone.

§ 11. Although the statistics of the New York banks go back for over forty years, it seemed best to begin with the year 1879 to avoid the results of the inflation years following the war.

The tables of the reserves and loans of the New York Associated Banks immediately precede. Table No. 3 contains the total reserves for each week of the twenty-two years, 1879–1900. The loans appear in Table No. 4. To find the exact date on which any week in any year ends, add or subtract (as the sign is plus or minus) the number given directly under the year at the head of the column from the date given in the second column corresponding to the numerical week desired.

For the reason that tables of raw figures are indigestible to the eye, the two tables are plotted on the Chart.† The horizontal axis denotes time. Vertically upon the Chart is plotted the amount of the total reserves in millions of dollars. The solid black line joining these values will be called from this point on the *Reserve Polygon*. The two upper irregular lines are the *Loan* and *Deposit Polygons* respectively. No statistical table of the deposits is given for the reason that the deposits have, as yet, been little studied. The time unit,

* Cf. the constant references in Sumner's *History of American Currency* to these statements from 1853–1876.

† See Appendix.

the week, is the same for the three polygons, and this facilitates comparison between movements of reserve, loan and deposit polygons. The vertical scale of the reserve polygon is twice that used in plotting the loan and deposit polygons. The three steadily ascending smooth curves will be explained in the next chapter.

§ 12. In such arrays of statistics, it is often convenient to resolve the motion of the polygons into elements. By element I mean an ideal influence which is at work in the curve in a certain motion correlate with time.

In these charts we may consider three influences at work or *elements* of the polygons. These are:—

(I) The *Growth Element*, i. e. growth in the fund due to a continuous passage of time;

(II) *Periodic Elements*, i. e. periodicity in the fund due to recurrent periods of time—as the year, the quarter, the month, the week or the day;

(III) *Dynamic Elements*, i. e. influences outside the previous two, which may be either of the nature of

(1) A *Cycle*, i. e. an uncertain period with gradual change,

(2) A *Catastrophe*, i. e. a crisis with violent change, or

(3) *Minor Dynamical Changes*.

The general problem is to isolate these elements and further to go beyond the time elements to the causes.

CHAPTER II

THE GROWTH ELEMENT

§ 13. A glance at the Chart discloses two facts, (1) that there has been an increase in all the items coincident with the passing of the years, and (2) that there is also a series of fluctuations up and down. Within the solar system the earth has two motions, revolution about the sun and rotation about its axis. The problems are roughly analogous.

Following the analysis of § 12, the growth element will be first discussed. The idea of growth is a familiar one in economics and needs no elaboration here. The Chart shows for twenty-two years a more or less steady advance with many temporary recessions. Considered rationally, the growth element should be a steadily ascending curve. In economics we have been accustomed to the use of the term geometrical increase. The geometrical curve is used by the census officials to calculate population statistics between decades. It is involved in all the popular notions of compound interest. Technically, the curve that will best fit the statistics is what we want. Practically, as we shall see, the simple geometrical curve furnishes a very satisfactory representation of the growth element.

§ 14. The practice of fitting smooth curves to statistics has been in vogue in economics for a considerable time. Pareto's income curve, the work of Edgeworth, and a score of other studies are well known.

What form of curve is best suited to the statistics is a question which always arises. Having determined which form to try, the mathematical curve may be determined by the method of interpolation.* This consists in solving a num-

* Professor Pearson's "method of moments," generalized for various forms, is promised at this time of writing. The results can hardly stop short of a revolution in this field of study.

ber of observational equations, which operation determines the constants of the form. There is an able work by Anton Steinhauser on interpolation.* In this work may be found a table† of sixteen convenient interpolation forms.

Each form has different properties, and Steinhauser uses the method of least squares to determine the form of closest fit. The method of least squares does very well for statistics that are subject to one dominating influence. But when, as in these polygons of the reserves, loans and deposits, the deviations from the interpolated form become continually more violent with the passage of time in some proportion to the fund, this method is of little use; for the later years exert

* Die Lehre von der Aufstellung Empirischer Formeln mit Hilfe der Methode der Kleinsten Quadrate. Leipzig, 1889.

† INTERPOLATION FORMS (STEINHAUSER)

No.	Relation X and Y.	Form.	Curve.
1	Reciprocal	$Y=A+B/(C-X)$	Hyperbola
2	Proportional	$Y=BX$	Straight line
3	Y—Arith. prog. 1st order X—Arith. prog. 1st order	$Y=A+BX$	Straight line
4	Y—Arith. prog. 2d order X—Arith. prog. 1st order	$Y=A+BX+CX^2$	Parabola
5	Y—Arith. prog. 3d order X—Arith. prog. 1st order	$Y=A+BX+CX^2+DX^3$	Parabolic curve
6	Y—Geom. prog. X—Arith. prog. 1st order	$Y=10^{(a+bx)}$ or $\text{Log } Y=A+BX$	Logarithmic curve
7	Y—Arith. prog. 1st order X—Geom. prog.	$Y=A+B \text{ Log } X$	Logarithmic curve
8	Y—Geom. prog. X—Geom. prog.	$Y=10^{a+b \log x}$ or $\text{Log } Y=A+B \text{ Log } X$	Exponential curve
9	Y—Arith. prog. 2d order X—Geom. prog.	$Y=A+B \text{ Log } X+C \text{ Log}^2 X$	No name
10	1st diff. Y geom. prog. X—Arith. prog. 1st order	$Y=A+BC^x$	Logarithmic curve
11	Y—Arith. prog. 1st order 1st diff. X geom. prog.	$Y=A+B \text{ Log } (X-C)$	Logarithmic curve
12	Y—Geom. prog. 1st diff. X geom. prog.	$\text{Log } Y=A+B \text{ Log } (X-C)$	No name
13	1st diff. Y geom. prog. X—Geom. prog.	$Y=A+BC^{\log x}$	No name
14	1st diff. Y geom. prog. 1st diff. X geom. prog.	$Y=A+B(CX+D)^e$	No name
15	mth root Y arith. prog. X—Arith. prog.	$Y=(A+BX)^c$	No name
16	1st diff. Y geom. prog. X—Arith. prog. 1st order	$\text{Log } Y=A+BX^c$ or $Y=10^{(a+bx^c)}$	No name

too great an influence in the determination of the probable variation.

§ 15. After a consideration of the practical difficulties as well as the needs of theory, it seemed best to use as a growth axis the geometrical curve* of simplest type $Y=BC^x$ (Steinhauser's form 10). This form is familiar in the algebraic formula for compound interest,

$$A=P(1+R)^t,$$

where A is the amount after T years, P the principal and R the rate. This form is probably not the best fit theoretically, but it is a good fit approximately, as we shall see, and to increase the number of constants was practically an added burden in arithmetical computation that was out of the question. Moreover, the convenience of the formula would be greatly lessened by greater complication and the gain would not be, perhaps, after all very apparent for our purposes.

So having once assumed the geometrical form $Y=BC^x$ as the growth axis, it is simply necessary to determine the constants B and C of the form $Y=BC^x$ for the three items of the bank statements, the total reserves, the loans and the deposits.

§ 16. Although the method of interpolating the geometrical curve is quite widely known in economics, for the sake of completeness it may be well to sketch roughly the procedure. The geometrical form $Y=BC^x$ when written in logarithms

$$\text{Log } Y=\text{Log } B+X \text{ Log } C,$$

becomes an arithmetical form, i. e. if plotted (Diagram No. 2 on the Chart), the logarithmic curve becomes a straight line.

Now it is obvious that, having collected the statistics of the three bank items for the years 1879–1900, we know the amount of each item for every week. If we represent by (Y) the amount of the item for a week which is (X) years from the first week of 1879, we may say that the observational values of (Y) and (X) in the above equation are known for every week of the twenty-two years. We do not know the

* The writer tried tentatively several modified forms by graphic processes.

values of the constants (B) and (C). Graphically (Diagram No. 2 on folding chart), Log B represents the point at which the straight line starts. Log C is the increase at the end of one year. To find these two unknown constants B and C, the following tables are prepared.

I.					II.				
Log Y (1)	=	Log B	+	X Log C (2)	X Log Y (3)	=	X Log B	+	X^2 Log C (4)
....		....		....	....		....		....
....		....		....	....		....		....
Σ Log Y	=	N Log B	+	Log CΣX	ΣX · Log Y	=	Log B ΣX	+	Log CΣX^2

In column (1) all observations for the twenty-two years are entered weekly. In column (2) is entered the time of the entry in column (1), counting from January 7, 1879. Columns (1) and (2) are multiplied through by the successive values of X, giving columns (3) and (4). Inasmuch as $X=0$ for the record of January 1, 1879, the first records vanish in (3) and (4), and so the first series of equations contains one more equation than the second series. The entries in each column are added and the sums form the two equations written at the foot of the tables. The solution of these two equations determines the two unknowns, Log B and Log C. The values of the constants B and C are, of course, determined by the logarithms and may now be inserted in the form $Y=BC^x$. This formula renders it possible to calculate for any week the value as given by the form. These values plotted form the steadily rising geometrical curve. This, in brief, is the statistical mechanism of the growth axis.

§ 17. In practical work, it is impossible to use every record. To do so would involve an unbearable amount of arithmetical computation. For instance, in each case we should have to add four columns of over 1100 numbers of four or five figures in addition to over 2200 multiplications of an average of seven figures by three. The work might be lessened by the use of multiplication tables and the adding machine, but the end would not justify the means.

It is enough to take readings at equal intervals, say monthly or bi-monthly. In some cases it may be well to

underweight years out of the ordinary as crises, but on the whole the impartial method of equal intervals is preferable; for too much work should not be put upon the growth axes, which are after all but the scaffoldings of the polygons.

§ 18. To illustrate the method of interpolation more fully, the GROWTH AXIS OF THE TOTAL RESERVES is in point. In the form $Y=BC^x$, Y equals the value of the total reserve, R_t, for any week, T, and may be written R_t. X represents the interval of time since January 1, 1879, for any week. B stands for the first value, R_0, and C stands for $(1+r)$, i. e. one plus the annual rate of increase. Writing the form $Y=BC^x$ in these new letters, we obtain the formula

$$R_t=R_0(1+r)^t.$$

Writing logarithms, we have

$$\text{Log } R_t=\text{Log } R_0+T \text{ Log } (1+r).$$

The two unknowns are, of course, B and C, i. e. Log R_0 and Log $(1+r)$. In words, it is necessary to know at what point to start the growth axis, and how rapidly to make it rise.

In the following work 136 records were taken. Constructing the two tables as stated in § 16, the two following equations were obtained on adding the four columns.*

$$\begin{aligned} 276.378877 &= 136 \text{ Log } R_0+ 1,216.2 \text{ Log } (1+r) \\ 2,599.298623 &= 1,216.2 \text{ Log } R_0+16,196.8 \text{ Log } (1+r) \end{aligned}$$

Solving,

$$\begin{aligned} \text{Log } (1+r) &= 0.023317 \\ \text{Log } R_0 &= 1.823682 \end{aligned}$$

The logarithmic equation of the growth axis is then

$$\text{Log } R_0=1.823682+0.023317 \text{ t}.$$

Dispensing with logarithms, we have approximately

$$R_0=66.63 \ (1.055)^t.$$

§ 19. Following the method of interpolation used in the determination of the growth axis of the reserve polygon,

* In actual work, it should be stated, there are several short cuts which considerably modify the burden of calculation.

the equation of the GROWTH AXIS OF THE DEPOSITS was found to be

$$Y = 279.8\ (1.0397)^t,$$

or expressed in logarithms,

$$\text{Log } Y = 2.446987 + 0.016926\ t.$$

This is based on 155 records at more or less even intervals with a slight weighting.

The equation of the GROWTH AXIS OF THE LOANS is based on 88 records. The equation is

$$L_t = 264.7\ (1.042)^t,$$

or expressed in logarithms,

$$\text{Log } L_t = 2.422717 + 0.017742\ t.$$

§ 20. These equations enable us to work out the values of the growth axes for any week of the twenty-two years. In the following table (No. 5) are the values of the growth axes of the reserves, loans and deposits on the first day of January in each year. The values in this table, when plotted on the Chart, form the "three steadily ascending smooth curves" mentioned in § 11. The lowest smooth curve is the growth axis of the reserves. Next above is the growth axis of the loans. The highest smooth curve represents the growth element of the deposits.

TABLE NO. 5.

Growth Axes Values for January 1 of each year, in millions of dollars.

Year.	Reserves.	Deposits.	Loans.	Year.	Reserves.	Deposits.	Loans.
1879	67	280	265	1891	127	447	432
1880	70	291	276	1892	134	465	450
1881	74	303	287	1893	141	483	469
1882	78	315	299	1894	149	502	489
1883	83	327	312	1895	157	522	509
1884	87	340	325	1896	166	543	530
1885	92	354	338	1897	175	565	552
1886	97	368	352	1898	185	587	575
1887	102	382	367	1899	195	610	599
1888	108	398	382	1900	206	635	624
1889	114	413	398	1901	217	660	650
1890	120	430	415	1902	229	686	677

§ 21. If now we turn to the Chart and consider how well each geometrical growth axis fits its respective polygon, the question arises what is meant by a 'good fit?' The popular notion would perhaps be that the polygon should balance as evenly as possible on its growth axis.

From the notion of an even balance the mathematical dogma has been developed that the minimizing of the squares of the deviations constitutes the criterion of a 'good fit.' In an ordinary arithmetical form—an example would be the expansion of mercury under heat—the deviations are in amplitude no greater with a higher temperature than with a lower one. In such a case the higher deviations exert the same influence in the determination of the probable variation as do the lower deviations, and in such cases of a constant law of deviation the quantitative test of least squares is of real value. But in the cases of an increasing or of a decreasing amplitude of deviations, this quantitative index is unrepresentative and so far as I can see a very bad test.

In this case of increasing funds in which the deviations become in amplitude constantly more violent, a very different notion of an even balance is involved. For in a continually increasing amplitude the amplitude might come, at last, to equal the whole fund so that the final squares would exert so vast an influence upon the sum of the squares, that the squares of the last few deviations would equal the sum of the squares of all the deviations that had gone before. In this extreme case, the meaning of the least square test is entirely lost.

It might seem best to interpolate the deviations without regard to signs and to obtain a ratio between the fund and the amplitude by a comparison of the growth axes of the fund and of the deviations. In this way, we might reduce all the deviations by this changing ratio to the fund and apply the least square test to these proportionate deviations. Such a method brings up a rather complex theory in probabilities. The difficulties in practical work would be also very great owing to the large amount of tentative arithmetical calculation.

Out of the original, popular notion of an even balance, another interpretation may be developed. Abandon all idea of a quantitative test of measured deviations, and rely upon a qualitative test. Practically what is wanted is a growth axis which shall all the time be crossed and recrossed by the polygon. The form, then, which shows a greater number of crossings with a higher evenness of distribution is the better form. This is a convenient method for the statistician. All that is required, in an example of this sort, is a table showing the number of crossings per year. Calculate the average number of yearly crossings with the probable variation, and this range becomes the test of fitness. The greater the average and the smaller the probable variation, the better is the form. It is not a decisive criterion; for sometimes the advantage between a slightly lower average with narrower range and a higher average with wider range would be difficult to decide. In such a case, simplicity of form should be the deciding factor for the statistician.

Table No. 6 records the number of times during each year that the reserve, loan and deposit polygons crossed their respective growth axes.

TABLE NO. 6.

Year.	Reserves.	Loans.	Deposits.	Year.	Reserves.	Loans.	Deposits.
1879	6	5	0	1891	1	0	0
1880	4	2	2	1892	1	2	2
1881	4	0	4	1893	3	0	3
1882	5	2	4	1894	0	0	0
1883	5	4	0	1895	3	0	2
1884	2	1	2	1896	0	0	0
1885	0	0	5	1897	5	3	5
1886	2	2	3	1898	2	2	0
1887	2	2	2	1899	1	0	0
1888	3	2	4	1900	5	0	0
1889	6	8	4				
1890	2	2	0	Average	2.8	1.5	1.9
				Range	1.2	1.0	1.2

Thus, during the twenty-two years, the reserve polygon crosses the growth axis 62 times, or on an average of 2.8 crossings per year. The deposit polygon crosses 42 times,

or on an average of 1.9 times yearly, and the loan polygon makes 33 crossings, with an average of 1.5 crossings yearly.

The probable annual range of the reserve polygon with respect to the number of crossings is 1.6 to 4.0, of the deposits 0.7 to 3.1, and of the loans 0.5 to 2.5. The method of calculating the probable range is doubtless familiar. It is briefly described at a later point.* The commonly accepted meaning is that in any year the chances are even that the number of crossings will lie within this range.

§ 22. One or two interesting mathematical properties of these growth axes may be remarked. The ratios of annual increase are for the reserve axis 5.5%, the deposit axis 4.0%, and the loan axis 4.2%, approximately. Now, owing to these varying rates, the ratios existing between the growth axes for any date must change. Thus, for the first week in 1879, the ratio of reserve axis to deposit axis is 23.8% and of reserve axis to loan axis 25.2%. Twenty-two years later, the ratio of reserve axis to deposit axis is 32.0%, and the ratio of reserve axis to loan axis is 32.5%. During the same years, the ratio of the loan axis to the deposit axis has increased from 94.6% to 98.2%.

Now these changes to a considerable extent represent the influences at work changing the relative amounts of the various funds. The rise of the ratio of reserve axis to loan axis from 25.2% to 32.5% is probably largely due to the diminished relative use of bank-note circulation based on bonds. A larger relative reserve in money is now required to take the place of the relatively diminished use of bank-notes.

The same tendency accounts for the increased ratio of the loan axis to the deposit axis. The ratio advances from 94.6% to 98.2%. But here too much stress should not be laid; for the growth axis and the loan axis are gradually converging. If we solve the logarithmic equation,

$$\text{Log } L_t = \text{Log } D_t$$
$$2.422717 + 0.017742\ t = 2.446987 + 0.016926\ t,$$

to discover when the axes will be equal, we shall find that

* Section 33.

they cross about 1910. Consequently, before results should be drawn from such interpolation, it would be advisable to construct the axes with greater care. For our purposes, however, as scaffolding, they are sufficiently accurate. The summary of the facts of the growth axes, as they are, will be found in Table No. 7.

TABLE No. 7.

	Reserves.	Loans.	Deposits.
Equation, ...	$66.6 (1.055)^t$	$264.7 (1.042)^t$	$279.8 (1.040)^t$
Log. Equation, . .	1.823682+0.023317 t	2.422617+0.017742 t	2.446987+0.016926 t
Ratio 1879, . . .	R/D 23.8%	L/D 94.6%	R/L 25.2%
Ratio 1900, . . .	32.0%	98.2%	32.5%
Crossings (1879–1900),	62	33	42
Average Crossings,	2.8	1.5	1.9
Probable Variation,	1.2	1.0	1.2

§ 23. The mathematical determination of these several growth axes naturally leads up to the question, 'To what use can they be put?'

It will be remembered that in a preliminary analysis, three main classes of elemental influences were assumed to be at work governing the values of the funds at any time. The three influences we called (1) the growth element, (2) the periodic elements, and (3) the dynamic elements. The growth elements have now been studied and mathematical laws derived to express their motion.

It has also been previously remarked that the three polygons are continually crossing and recrossing their respective growth axes. At this point, we may bring in a new working assumption,—that the fluctuations or deviations from the growth axes during these years are due to other influences at work than the growth element. It remains to discover whether the results justify the use of this assumption. It is thus that we are enabled to eliminate the element of growth.

A study of the folding Chart discovers two facts:

(1) That the growth axes afford an excellent standard from which to measure the deviations of the polygons at any date;

(2) That these deviations increase in amplitude as the values of the funds swell.

The growth axis as a standard from which to measure deviations is analogous to the "ideal" surface of the ocean in another science. Nor is the analogy lost in (2)—that the deviations increase with the increasing value of the funds. For the waves that ruffle the surface of a fish globe are in amplitude much smaller than the swell of the Atlantic Ocean. Whether the deviations are fairly represented by a percentage proportion, will be considered later. In the following pages I have assumed a percentage proportion and have reduced the actual deviations to a percentage basis.

With all the criticism that exists in regard to governmental statistics, they nevertheless remain of very great value. For although the final sums, after forty years of additions and subtractions of four and five estimated amounts annually, is subject, undoubtedly, to very great error, still it often becomes difficult to conceal the tendencies in the changes from year to year.

It is with these changes that interesting problems in economics are connected, not with gross sums. Like the sailor or captain on the ocean, we are interested, not in the depth below of the ocean-bed,—for we are sure that the ocean-bed will remain during the little while that we are sailing over it,—but we do fear the height of the waves upon the surface. Even the captain measures the height of the waves, not from the ocean-bed, but from the "ideal" surface. So with us,—we will measure the financial waves from an "ideal" standard, which for convenience we have called the growth axis.

Nor are these waves unimportant. It was some chemist who said, "Never throw away your residues. Look in these for your results." This practice of studying the residues is perhaps most common in the business world. A glance through the tables of trade and financial statistics compiled by our leading journals will convince the reader how familiar this method is outside the text-books. The mill owner and the speculator are constantly watching the net changes, the differences, and these become the motives of their actions. The economist may profitably study these differences for the verification of his laws.

CHAPTER III

THE PERCENTAGE DEVIATIONS

§ 24. The *Percentage Deviation* for any week may be defined as the ratio of the algebraic difference of the actual fund and the growth axis for the given week to the growth axis. Let MN (Diagram No. 3 on the Chart) be the growth axis and PAQ the polygon. If AB is the actual deviation, BC the growth axis for the actual fund AC, then AB/BC is the percentage deviation.* As the polygon lies above or below the growth axis, the sign is positive or negative.

It is thus possible, given the requisite *data*, to construct a table of percentage deviations. In such a table the element of growth and the increasing amplitude of deviations (§ 21) are eliminated from the statistics. These percentage deviations may then be plotted upon a horizontal axis, which is the growth axis reduced to a straight line.† The rising polygon, then, becomes undulatory in motion, similar in type to curves of fluctuations of the daily average temperature, the foreign exchange rate, etc. It is necessary to reduce the actual polygons to an undulatory motion to satisfactorily study correlation with other undulatory curves.

§ 25. Following the method laid down in the last section, I have calculated the weekly percentage deviations of reserves, loans and deposits for the twenty-two years. The weekly percentage deviations of the total reserves (which for brevity we will call, hereafter, *reserve deviations*), may be consulted in Table No. 8.‡ The weekly percentage deviations of the

* The percentage deviation (D) for the geometrical growth axis may be algebraically expressed as follows:—

$$D=\frac{AC-BC}{BC}=\frac{R_{t1}-R_{tg}}{R_{tg}}=\frac{R_{t1}-R_0(1+r)^t}{R_0(1+r)^t}.$$

R_{tg} represents the value of the growth axis for the time (t).

† Cf. Mercator's Chart for a parallel projection.

‡ In Tables Nos. 8 and 9 fractions of 1% are expressed as the nearest whole percentage. In the actual calculation, however, of all following statistical tables, fractions were expressed as the nearest tenths of 1%.

loans (i. e. the *loan deviations*), appear in Table No. 9. The *deposit deviations* are omitted for precisely the same reason that was given for the omission of the actual deposits.

Reserve, loan and deposit deviations are represented in the large Chart. The heavy straight line, in the lower half of the Chart, stands for the growth axes of reserves, loans and deposits reduced to a straight line. This line is zero value for the percentage deviations, which are plotted from this line as origin.

§ 26. In this Chart are represented the really important movements in these financial statistics. The motion of the growth element is slow and gradual. Its effect is scarcely felt. But in the deviations are the movements which are forever puzzling financiers, and upon whose often apparently eccentric movements great fortunes are made or wrecked, panics are bred and crises precipitated. These deviations do not, it is hardly necessary to say, produce such serious calamities as crises; but they are the barometers of the state of that conglomeration of many tendencies in the societary circulation, working for good or for ill, that are in themselves prosperity or depression. Indeed, they may be made to form a measure of the severity of crises or of the affluence of periods of prosperity, as we shall later see.

§ 27. Before commenting upon certain features suggested to the eye by the movements of the three polygons of the Chart, it is, perhaps, at this point the best place to explain what is meant by the *weekly changes* in the percentage deviations. The changes in the percentage deviations are simply the increases (or decreases) of the consequent week over the antecedent week. There are several advantages in working with the first differences instead of with the percentage deviations. Among these are the following:—

(1) The changes are smaller in amount and consequently much less arithmetical work is involved in calculating averages.

(2) The changes are easily found directly from the weekly report, and afford a convenient method of continuing a table

of percentage deviations. The actual change as reported by the papers is of course the distance AE in Diagram* No. 4. Now the change between the percentage deviations of the two weeks would be $\left(\frac{AC}{BC}-\frac{A'C'}{B'C'}\right)$. But $BC=[(1+r)^{1/52}\times B'C']$. It follows that $\frac{AC}{BC}-\frac{A'C'}{BC/(1+r)^{1/52}}=\frac{AC-A'C'(1+r)^{1/52}}{BC}$. Thus the change (C) (it may be shown† by a few steps), is given by the formula

$$C=\frac{R_2-R_1(1+r)^{1/52}}{R_{g_2}},$$

where R_2 is the actual reserve for the second week, R_1 for the first week, r the annual rate of increase of the growth axis, and R_{g_2} the value of the growth axis for the second week. For the growth axis of the reserves, $(1+r)^{1/52}$ equals the constant (1.0013). Often, for practical purposes, it is sufficiently accurate to divide the actual change as reported in the statement by the value of the growth axis for that date. But if it is planned to carry along a table of the percentage deviations by adding and subtracting the changes according to the signs, it is better to follow this rule.

To find the weekly change in the reserve deviations, subtract the product of the actual value for the previous week multiplied by the constant 1.0013 from the value for the present week and divide by the value of the growth axis for the present week. Add the change to the percentage deviation for the previous week,—or subtract if the change is negative. The result is the value of the percentage deviation for the present week. The values of the growth axis are, of course, found from a previously prepared table.

* See folding Chart.

† $\frac{AC-A'C\,(1+r)^{1/52}}{BC}=\frac{R_2-R_{g2}-(R_1-R_{g1})(1+r)^{1/52}}{R_{g2}}$

$$=\frac{R_2-R_1(1+r)^{1/52}-R_{g2}+R_{g1}(1+r)^{1/52}}{R_{g2}}=\frac{R_2-R_1(1+r)^{1/52}}{R_{g2}}.$$

For $R_{g2}=R_{g1}(1+r)^{1/52}$.

TABLE NO. 8.—WEEKLY PERCENTAGE DEVIATIONS OF THE TOTAL RESERVES.

Numerical Week.	Corresponding Month and Day.	1879	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
		− 3	+ 3 From March on + 2	− 3	0	− 1	− 2 From March on − 3	+ 3	+ 2	− 1	0 From March on − 1	− 2	− 3	+ 3	+ 2 From March on + 1	0	− 1	− 2	− 3 From March on − 4	+ 2	+ 1	0	− 1
1	Jan. 7	− 6	− 7	+ 2	0	− 3	+ 5	+47	+26	+ 5	− 4	− 4	−14	−12	+ 4	−13	+44	+11	−14	+ 3	+ 5	+18	+ 2
2	14	− 5	− 2	+ 9	+ 8	+ 3	+14	+52	+32	+ 9	+ 1	+ 3	− 9	− 6	+10	− 9	+51	+15	−10	+ 9	+ 7	+22	+ 5
3	21	+ 2	− 2	+12	+12	+ 4	+20	+52	+33	+11	+ 7	+ 7	− 7	− 1	+17	0	+56	+18	− 6	+13	+13	+27	+10
4	28	+ 7	− 2	+12	+12	+ 6	+23	+54	+36	+15	+ 9	+11	0	0	+22	+ 3	+63	+18	− 4	+16	+17	+31	+15
5	Feb. 4	+ 8	− 2	+12	+ 9	+ 3	+24	+56	+35	+14	+ 9	+11	+ 1	− 2	+21	0	+76	+10	− 2	+12	+18	+32	+17
6	11	+ 3	+ 1	+12	+ 4	+ 2	+28	+52	+36	+13	+ 7	+ 7	− 2	− 3	+22	− 1	+47	+ 6	− 2	+10	+17	+32	+18
7	18	− 1	+ 7	+ 8	− 1	− 2	+27	+49	+32	+10	+ 5	+11	− 5	− 6	+22	− 5	+32	+ 5	− 4	+13	+13	+32	+18
8	25	− 6	+ 1	0	− 7	− 5	+27	+48	+25	+ 5	+ 2	+10	− 9	− 9	+19	−10	+32	+ 3	−10	+14	+10	+32	+15
9	Mar. 4	−12	0	− 9	−11	−10	+24	+47	+23	+ 2	+ 2	+ 6	−12	−11	+15	−14	+32	+ 1	−12	+13	+ 6	+29	+12
10	11	−14	− 2	− 8	− 9	−16	+16	+47	+21	+ 1	− 3	+ 3	−15	−12	+11	−18	+32	− 2	−13	+12	+ 5	+27	+ 8
11	18	−16	− 6	− 4	− 5	−21	+ 8	+47	+16	0	− 4	+ 2	−14	−12	+12	−18	+42	− 7	−14	+ 8	+ 7	+24	+ 5
12	25	−18	− 7	− 6	− 5	−23	+10	+47	+ 8	− 3	− 5	+ 2	−12	−12	+12	−16	+43	−11	−16	+ 8	+10	+24	+ 4
13	April 1	−22	−11	− 6	− 8	−21	+ 8	+45	+ 5	− 6	− 6	− 1	−12	−12	+11	−16	+47	−12	−17	+ 7	+10	+22	+ 7
14	8	−26	−11	− 3	−10	−20	+ 2	+47	+ 8	− 5	− 7	− 6	−14	−14	+ 9	−17	+46	−13	−18	+ 6	+ 8	+21	+ 8
15	15	−18	−11	+ 2	− 3	−15	+ 2	+49	+ 9	− 5	− 5	− 1	−15	−16	+10	−15	+46	−12	−18	+ 7	+ 9	+22	+11
16	22	−12	−10	+ 8	+ 4	−13	− 1	+52	+ 8	− 5	− 2	+ 6	−15	−15	+13	−13	+46	− 8	−18	+ 9	+11	+24	+14
17	29	− 6	− 7	+13	+ 7	−12	− 2	+54	+ 5	− 3	+ 1	+ 7	−15	−15	+14	−16	+50	− 4	−16	+ 9	+11	+24	+16
18	May 6	0	− 1	+18	+ 7	− 9	− 5	+55	+ 4	− 5	+ 4	+ 3	−14	−18	+ 9	−16	+50	0	−13	+ 5	+11	+23	+18
19	13	+ 6	+ 5	+25	+ 5	− 3	− 1	+58	+ 5	− 6	+ 9	+ 3	−16	−19	+11	−11	+49	+ 3	−15	+ 5	+14	+27	+17
20	20	0	+ 9	+29	+ 2	− 3	− 6	+60	+ 5	− 6	+14	+ 8	−16	−20	+13	−14	+48	+10	−15	+ 6	+17	+31	+17
21	27	− 9	+14	+28	− 2	+ 3	−24	+61	+ 5	− 6	+15	+ 8	−14	−19	+17	− 6	+46	+13	−14	+ 7	+21	+34	+19
22	June 3	−11	+15	+24	− 2	+ 3	−21	+61	+ 3	− 7	+12	+ 8	−13	−21	+17	−11	+46	+14	−13	+ 7	+21	+29	+22
23	10	− 9	+19	+25	0	+ 5	−19	+64	+ 6	− 7	+14	+ 4	−13	−18	+17	−17	+45	+13	−16	+ 8	+24	+33	+21
24	17	− 7	+20	+24	+ 6	+ 5	−13	+64	+ 8	− 7	+17	+ 4	−12	−12	+17	−23	+44	+12	−14	+10	+29	+29	+21
25	24	− 7	+21	+25	+ 7	+ 7	−10	+66	+ 8	− 9	+17	+ 2	−13	− 8	+13	−27	+44	+12	−16	+11	+30	+26	+19
26	July 1	− 3	+24	+23	+ 2	+ 7	− 4	+65	+ 5	− 9	+17	+ 1	−12	−10	+11	−30	+43	+10	−15	+ 7	+32	+21	+19

TABLE NO. 8 (CONTINUED).—WEEKLY PERCENTAGE DEVIATIONS OF THE TOTAL RESERVES.

Numerical Week.	Corresponding Month and Day.	1879	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
		− 3	+ 3 From March on + 2	− 3	0	− 1	− 2 From March on − 3	+ 3	+ 2	− 1	0 From March on − 1	− 2	− 3	+ 3	+ 2 From March on + 1	0	− 1	− 2	− 3 From March on − 4	+ 2	+ 1	0	− 1
27	July 8	+ 1	+25	+24	+ 9	+ 3	0	+68	+ 7	− 6	+16	− 1	−13	−11	+ 9	−35	+44	+ 8	−16	+ 8	+28	+15	+18
28	15	+ 4	+26	+28	+13	+ 5	+12	+70	+ 8	− 5	+19	0	−11	− 7	+10	−35	+45	+ 8	−13	+11	+26	+16	+20
29	22	+ 3	+23	+28	+10	+ 8	+17	+68	+ 8	− 5	+17	0	−12	− 7	+12	−33	+45	+12	−13	+13	+20	+15	+22
30	29	+ 8	+22	+28	+ 6	+ 7	+20	+68	+ 6	− 7	+17	− 1	−12	− 7	+14	−37	+41	+14	−20	+12	+19	+13	+24
31	Aug. 5	+13	+18	+21	+ 3	+ 6	+19	+66	+ 2	− 9	+17	+ 1	− 9	− 9	+10	−45	+40	+14	−19	+ 9	+20	+10	+25
32	12	+ 2	+17	+15	+ 4	+ 7	+21	+63	− 2	−12	+15	− 3	−11	− 9	+ 7	−47	+38	+12	−22	+ 8	+17	+13	+25
33	19	− 7	+12	+ 8	+ 1	+ 3	+22	+63	− 5	−14	+13	− 6	−20	−12	+ 4	−44	+40	+14	−26	+10	+15	+14	+21
34	26	−11	+11	+ 2	− 1	0	+21	+60	− 6	−15	+11	− 9	−24	−13	+ 1	−41	+38	+11	−28	+10	+15	+13	+23
35	Sept. 2	−12	+10	+ 1	− 9	0	+20	+56	− 8	−14	+ 7	− 7	−23	−16	− 3	−37	+38	+12	−29	+ 7	+ 6	+10	+25
36	9	−14	+10	+ 3	− 9	− 2	+18	+52	− 7	−15	+ 2	− 4	−23	−16	− 6	−34	+41	+10	−29	+ 3	− 1	+ 5	+25
37	16	−14	+10	+ 3	− 8	− 2	+16	+49	− 7	−16	+ 2	− 6	−26	−17	− 8	−28	+34	+ 4	−29	− 2	− 5	+ 2	+22
38	23	−10	+ 7	+ 4	−12	− 5	+16	+46	− 5	−14	+ 1	−10	−19	−19	−10	−22	+34	+ 1	−29	− 5	− 5	0	+19
39	30	−13	+ 8	− 4	−14	− 7	+16	+43	− 9	−10	+ 5	−11	− 7	−21	−11	−16	+34	− 2	−27	− 6	0	− 2	+15
40	Oct. 7	−16	+10	− 8	−12	− 6	+19	+42	− 9	−10	+ 2	−14	− 8	−18	−14	−12	+34	− 7	−25	− 8	+ 3	− 3	+11
41	14	−15	+ 7	−10	−10	− 5	+23	+38	−10	−10	+ 2	−15	−16	−15	−16	− 6	+34	−10	−26	− 8	+ 5	− 4	+ 8
42	21	−15	+ 7	− 5	− 9	− 9	+23	+31	− 9	−11	+ 9	−14	−20	−12	−16	+ 1	+36	−10	−27	− 3	+ 9	− 5	+ 5
43	28	−17	+ 8	− 1	−11	−10	+22	+28	− 8	− 5	+ 7	−15	−20	−12	−15	+ 7	+37	− 9	−27	− 1	+18	− 5	+ 6
44	Nov. 4	−16	+ 7	− 2	−12	−10	+22	+24	− 9	− 7	+ 5	−15	−20	−16	−17	+11	+37	− 9	−26	− 3	+ 9	− 7	+ 6
45	11	−18	+ 5	− 3	−18	− 8	+26	+25	− 6	− 9	+ 2	−16	−24	−13	−17	+10	+35	− 8	−28	− 1	+ 8	−10	+ 4
46	18	− 6	+ 3	− 3	−15	− 5	+31	+26	− 3	−10	+ 2	−15	−24	−10	−16	+23	+36	− 8	−21	0	+11	−10	+ 6
47	25	− 1	− 3	− 6	−18	− 2	+34	+25	− 4	−12	+ 2	−15	−24	− 8	−14	+29	+38	− 9	−15	+ 2	+11	− 6	+10
48	Dec. 2	− 1	−10	− 9	−13	0	+36	+23	− 6	−12	− 1	−15	−24	− 8	−14	+35	+26	−10	−11	+ 3	+12	− 4	+11
49	9	− 1	−10	−10	− 8	− 1	+37	+26	− 9	−13	− 4	−16	−27	− 6	−16	+36	+12	− 9	− 9	+ 2	+12	− 5	+ 8
50	16	− 4	− 7	− 9	− 5	− 1	+37	+24	−10	−11	− 3	−15	−25	− 1	−16	+36	+12	−10	− 8	+ 1	+13	− 6	+ 6
51	23	−10	− 5	− 6	− 5	− 1	+36	+22	−10	−10	− 5	−14	−21	0	−17	+38	+10	−12	− 5	− 3	+15	− 4	+ 7
52	30	−13	− 5	− 6	− 8	0	+36	+21	− 9	− 9	− 6	−15	−18	+ 5	−16	+40	+10	−15	− 5	0	+15	− 4	+10

TABLE No. 9—WEEKLY PERCENTAGE DEVIATIONS OF THE LOANS.

Numerical Week.	Corresponding Month and Day.	1879	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
		− 3	+ 3 From March on + 2	− 3	0	− 1	− 2 From March on − 3	+ 3	+ 2	− 1	0 From March on − 1	− 2	− 3	+ 3	+ 2 From March on + 1	0	− 1	− 2	− 3 From March on − 4	+ 2	+ 1	0	− 1
1	Jan. 7	−12	0	+ 6	+ 7	+ 2	+ 2	−13	− 4	− 5	− 6	− 2	− 4	−11	− 1	− 6	−14	− 3	−11	−11	+ 6	+19	+ 9
2	14	−13	0	+ 5	+ 7	+ 2	+ 2	−13	− 4	− 5	− 7	− 3	− 4	−11	− 1	− 6	−15	− 4	−11	−11	+ 6	+20	+ 8
3	21	−12	+ 1	+ 7	+ 7	+ 2	+ 2	−13	− 4	− 4	− 7	− 2	− 3	−11	− 1	− 5	−14	− 4	−11	−11	+ 8	+20	+ 9
4	28	−12	+ 2	+ 8	+ 8	+ 1	+ 3	−14	− 5	− 4	− 7	− 2	− 4	−10	0	− 5	−15	− 4	−12	−12	+ 9	+21	+10
5	Feb. 4	−10	+ 4	+10	+10	+ 1	+ 4	−13	− 4	− 3	− 6	0	− 3	− 8	+ 2	− 3	−14	− 4	−10	−10	+10	+23	+12
6	11	− 9	+ 4	+10	+ 9	+ 3	+ 5	−12	− 3	− 1	− 5	+ 2	− 1	− 7	+ 3	− 1	−12	− 5	−10	−10	+11	+25	+15
7	18	− 8	+ 4	+11	+ 9	+ 3	+ 6	−13	− 2	0	− 5	+ 2	− 1	− 7	+ 5	− 2	−11	− 5	−10	−10	+12	+26	+17
8	25	− 8	+ 6	+10	+ 8	+ 4	+ 6	−12	− 1	0	− 6	+ 2	− 1	− 7	+ 6	− 3	−10	− 6	−11	−11	+11	+28	+19
9	Mar. 4	− 7	+ 7	+ 3	+ 7	+ 4	+ 6	−11	0	0	− 5	+ 3	− 1	− 7	+ 8	− 4	−11	− 5	−10	−10	+ 9	+29	+20
10	11	− 7	+ 6	+ 2	+ 4	+ 4	+ 7	−11	+ 1	0	− 4	+ 4	− 3	− 7	+ 9	− 6	−11	− 5	− 9	− 9	+ 7	+29	+21
11	18	− 8	+ 5	+ 4	+ 4	+ 2	+ 7	−12	+ 1	0	− 4	+ 5	− 3	− 6	+ 9	− 7	−10	− 5	− 9	− 9	+ 5	+28	+20
12	25	− 9	+ 4	+ 4	+ 3	− 1	+ 6	−12	0	− 1	− 4	+ 5	− 4	− 6	+ 8	− 9	−10	− 6	−10	−10	+ 3	+29	+17
13	April 1	−10	+ 4	+ 4	+ 4	− 1	+ 6	−11	− 2	− 1	− 5	+ 5	− 4	− 5	+ 8	− 9	−10	− 6	−10	−10	+ 3	+29	+18
14	8	−12	+ 3	+ 5	+ 4	− 1	+ 6	−12	− 2	0	− 5	+ 4	− 3	− 5	+ 8	− 8	− 9	− 7	−10	−10	+ 1	+29	+20
15	15	−14	+ 1	+ 2	+ 3	− 2	+ 6	−12	− 2	− 1	− 5	+ 4	− 3	− 6	+ 8	− 9	− 8	− 7	−10	−10	0	+27	+21
16	22	−14	0	+ 5	+ 2	− 1	+ 6	−13	− 2	− 2	− 6	+ 4	− 4	− 7	+ 8	−10	− 7	− 7	−10	−10	− 2	+26	+21
17	29	−14	0	+ 5	+ 3	− 1	+ 5	−14	− 2	− 3	− 6	+ 3	− 5	− 8	+ 8	−10	− 7	− 7	−10	−10	− 2	+25	+22
18	May 6	−11	0	+ 5	+ 4	0	+ 4	−13	− 2	− 2	− 6	+ 4	− 5	− 8	+ 8	−10	− 6	− 6	−10	−10	− 2	+28	+24
19	13	−10	− 1	+ 9	+ 4	0	+ 1	−13	− 3	− 2	− 6	+ 3	− 4	− 9	+ 8	− 9	− 6	− 5	−12	−10	− 2	+27	+24
20	20	− 6	− 3	+11	+ 4	0	− 1	−14	− 4	− 2	− 7	+ 2	− 5	−11	+ 7	−12	− 6	− 4	−12	−10	0	+26	+24
21	27	− 4	− 3	+14	+ 5	0	− 5	−15	− 5	− 2	− 6	+ 2	− 5	−12	+ 7	−13	− 6	− 3	−12	−10	− 1	+23	+25
22	June 3	− 4	− 2	+17	+ 5	0	− 6	−14	− 4	− 2	− 7	+ 2	− 6	−12	+ 8	−13	− 6	− 3	−12	− 9	+ 3	+23	+26
23	10	− 4	− 1	+19	+ 5	+ 1	− 9	−14	− 4	− 2	− 6	+ 2	− 6	−13	+ 9	−13	− 6	− 3	−12	− 9	+ 4	+24	+27
24	17	− 5	+ 1	+19	+ 4	+ 2	−10	−13	− 3	− 2	− 5	+ 3	− 6	−12	+ 8	−14	− 6	− 2	−12	− 8	+ 4	+27	+28
25	24	− 5	+ 1	+18	+ 5	+ 3	−11	−12	− 2	− 2	− 4	+ 3	− 6	−12	+ 8	−15	− 6	− 1	−12	− 7	+ 5	+28	+28
26	July 1	− 6	+ 3	+20	+ 6	+ 3	−11	−11	− 1	− 3	− 3	+ 3	− 6	−11	+ 8	−14	− 6	− 1	−12	− 6	+ 6	+29	+27

TABLE NO. 9 (CONTINUED).—WEEKLY PERCENTAGE DEVIATIONS OF THE LOANS.

Numerical Week.	Corresponding Month and Day.	1879	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
		− 3	+ 3 From March on + 2	− 3	0	− 1	− 2 From March on − 3	+ 3	+ 2	− 1	0 From March on − 1	− 2	− 3	+ 3	+ 2 From March on + 1	0	− 1	− 2	− 3 From March on − 4	+ 2	+ 1	0	− 1
27	July 8	− 5	+ 3	+21	+ 7	+ 3	−10	−11	− 1	− 4	− 3	+ 4	− 4	−11	+ 7	−13	− 3	− 1	−12	− 5	+ 8	+30	+26
28	15	− 3	+ 3	+19	+ 6	+ 4	−13	−11	− 2	− 4	− 3	+ 4	− 5	−11	+ 5	−14	− 3	− 2	−12	− 5	+ 9	+27	+26
29	22	− 3	+ 4	+19	+ 8	+ 3	−13	−11	− 2	− 5	− 3	+ 3	− 5	−12	+ 4	−15	− 3	− 3	−12	− 4	+ 8	+25	+25
30	29	− 3	+ 5	+19	+ 9	+ 3	−13	−12	− 2	− 5	− 3	+ 2	− 6	−12	+ 5	−15	− 4	− 3	−13	− 4	+ 8	+24	+26
31	Aug. 5	− 1	+ 8	+19	+ 9	+ 3	−13	−10	− 1	− 5	− 3	+ 1	− 6	−12	+ 6	−15	− 4	− 2	−14	− 3	+10	+23	+26
32	12	+ 1	+ 8	+19	+10	+ 2	−13	− 9	− 2	− 6	− 2	+ 2	− 5	−11	+ 6	−14	− 3	− 2	−14	− 2	+12	+22	+26
33	19	+ 1	+10	+19	+10	+ 3	−14	− 9	− 7	− 7	− 1	+ 2	− 6	−11	+ 7	−16	− 3	− 2	−15	− 1	+13	+22	+27
34	26	− 3	+10	+17	+ 9	+ 2	−13	− 8	− 7	− 7	− 1	− 1	− 7	−11	+ 6	−16	− 3	− 2	−16	− 1	+14	+23	+28
35	Sept. 2	− 5	+10	+14	+ 8	+ 2	−14	− 7	− 8	− 8	0	0	− 8	−10	+ 5	−17	− 2	− 2	−16	0	+14	+22	+27
36	9	− 5	+10	+13	+ 7	+ 2	−13	− 6	− 8	− 8	0	0	− 7	−10	+ 4	−18	− 2	− 1	−17	+ 2	+13	+21	+28
37	16	− 6	+10	+13	+ 6	+ 3	−13	− 6	− 7	− 8	0	0	− 8	− 9	+ 3	−19	− 2	0	−17	+ 2	+10	+20	+29
38	23	− 5	+ 9	+13	+ 6	+ 3	−13	− 6	− 7	− 8	− 1	0	− 8	− 8	+ 1	−19	− 1	− 1	−17	+ 1	+ 8	+17	+28
39	30	− 4	+ 9	+12	+ 4	+ 3	−13	− 5	− 6	− 9	− 1	0	− 8	− 9	0	−19	− 1	− 3	−17	+ 1	+ 7	+16	+27
40	Oct. 7	− 2	+10	+10	+ 2	+ 2	−13	− 5	− 5	− 8	0	− 1	− 6	−10	0	−19	− 1	− 3	−17	0	+ 7	+15	+27
41	14	− 2	+11	+ 8	+ 1	+ 2	−13	− 4	− 6	− 7	+ 1	− 2	− 5	− 9	− 1	−18	− 1	− 4	−16	0	+ 9	+14	+25
42	21	− 2	+11	+ 5	0	+ 2	−13	− 3	− 6	− 7	0	− 3	− 5	− 9	− 3	−19	− 1	− 4	−17	− 2	+11	+13	+24
43	28	− 2	+11	+ 4	+ 1	+ 1	−13	− 2	− 6	− 8	0	− 4	− 6	− 9	− 4	−18	− 1	− 5	−18	− 1	+12	+12	+23
44	Nov. 4	− 1	+13	+ 6	+ 3	0	−13	− 3	− 7	− 7	0	− 4	− 7	− 8	− 4	−17	− 1	− 5	−19	0	+14	+12	+23
45	11	− 1	+13	+ 5	+ 2	0	−14	− 3	− 6	− 7	− 1	− 4	− 7	− 9	− 4	−17	− 1	− 6	−19	+ 1	+16	+11	+22
46	18	− 2	+10	+ 6	+ 1	0	−13	− 3	− 6	− 7	− 1	− 4	− 9	− 9	− 5	−17	− 1	− 7	−19	+ 1	+16	+10	+22
47	25	+ 1	+10	+ 7	0	0	−14	− 3	− 5	− 7	− 1	− 4	−10	− 8	− 5	−17	− 2	− 7	−17	+ 4	+16	+ 9	+23
48	Dec. 2	0	+ 6	+ 6	− 2	+ 1	−15	− 4	− 4	− 7	− 1	− 4	−11	− 7	− 5	−16	− 2	− 7	−14	+ 4	+17	+10	+24
49	9	− 1	+ 2	+ 6	− 2	+ 1	−15	− 4	− 4	− 7	− 2	− 5	−10	− 7	− 5	−15	0	− 7	−14	+ 6	+18	+10	+24
50	16	0	+ 2	+ 5	− 1	+ 1	−14	− 4	− 5	− 8	− 3	− 6	−11	− 6	− 6	−15	0	− 5	−12	+ 6	+19	+ 9	+22
51	23	− 1	+ 1	+ 5	− 1	+ 1	−13	− 4	− 6	− 8	− 3	− 5	−10	− 5	− 7	−15	− 2	− 8	−12	+ 6	+19	+ 8	+21
52	30	− 1	+ 2	+ 6	0	+ 2	−13	− 4	− 6	− 7	− 2	− 5	−11	− 3	− 7	−15	− 3	−10	−12	+ 6	+20	+ 8	+23

§28. To return to the Chart showing the reserve, loan and deposit deviations, several general features may be at once remarked:—

(1) With respect to violence of fluctuation, the reserve deviations surpass both in extent of deviation and in number of fluctuations. The deposit deviations are second in these respects, and the loan deviations third. Although the explanation will come out more fully later, the great deviating activity in the reserves is due to the deposit of currency in the New York banks by the other national banks of the United States as well as by the trust companies. The surplus cash of banks outside New York will draw interest if deposited in New York. When a use arises for this surplus cash, the banks in the West and South withdraw their balances, producing a descending fluctuation. This is one important cause. Another is found in the field of Foreign Exchange,—in the export and import of specie.

The deposits are second in this respect, because they stand on one side of the balance sheet offsetting reserves and loans. Fluctuations in reserves consequently produce fluctuations in deposits. The loan deviations are most lethargic owing to the fact that loans exist in response to business demands that are of a constant nature. Time is specified in the majority of the loan contracts, and this contract-time makes for greater stability.

(2) The leader in the up and down movement with respect to time is unquestionably the reserve. It begins ascending and begins descending in advance of the deposits and the loans. The deposits follow closely, and the loans follow shortly afterwards. There seems on the average about a three weeks interval between the turning points of the reserve and loan deviations. This is, of course, in consonance with banking theory, that expanding credits follow closely expanding reserves, and contracting reserves are followed by contracting credit. In this connection is the interesting study of pool activity in the flotation of new securities.

(3) The position of these lines with respect to the horizontal axis is an index of business prosperity. The lean years are the years in which the loan line is far below the horizontal axis with (as we shall see) interest low, and the fat years vice versa. For when more men can borrow more money at higher interest, then the profits of business are greater.

(4) This chart suggests strikingly the similitude of crises; but as this subject belongs under the dynamic element, discussion will be deferred.

(5) For purposes of comparison, there exists a real value in such a series of statistics. The removal of the growth element admits of comparison of the position of the fund at any time with respect to its position at any other time, by means of simple indices.

Such a method would, it seems to me, be of very great value if applied to the clearing statistics as published weekly by the Commercial and Financial Chronicle. The percentage of increase or of decrease should be calculated by the method of percentage deviations from a growth axis, and not, as now, from the same week of the previous year. The latter method does not admit of valid comparison; for a 100% increase after a year of great stagnation may be in reality far less than a 50% decrease from a year of great activity. The results of the two systems may be seen by the following table for the week ending July 1 in three years.

Year.	Actual Value.	Growth Axis.	Percentage Deviation.	Change in Percent. Dev.	Chronicle Change.
1887	40	50	−20%		
1888	90	60	+50%	+70%	+125%
1889	50	71	−29%	−79%	− 44%

How the rational change is distorted appears when we compare +70% with +125% or −79% with −44%.

In addition to the fallacy involved in the divisor, there arises another serious one. In some statistics there is no more reason for using as a basis of comparison the same week a year ago than for using any other week in past time;

for there often occurs no annual period, but a cycle of several years duration which is of great statistical importance. In such a case the rationale of the divisor entirely vanishes.

It seems probable that, could growth axes come into general use, the statistics published by the journals would be of much greater service to statistician, economist and business man. For they could then be conveniently used to study correlation with other sets of statistics, like the rate of foreign exchange or different varieties of the interest rate, all of which have an undulatory rhythm.

CHAPTER IV

PERIODICITY IN THE RESERVE DEVIATIONS

§ 29. In the preceding section, the growth element has been our main concern. We now take up the subject of periodicity. Periodicity is a very ancient notion in economics. It has remained a notion in economics more than in many of the other sciences.

A farmer's boy reading his almanac frames a concept of periodicity, if he has not already derived this notion from his own experience. In the almanac, he finds the lunar period, the solar period, the period of the tides and the period of the temperature. The almanac maker, however, has carried the notion beyond the hazy stages of common experience to the stage of statistical measurements and, finally, has derived tables so accurate as to admit of prediction. Why this should not become eventually possible in economics has never yet been clearly shown. Such a result, however, can come only through the statistical half of economics.

It should be remembered that time in itself has no causal connection with periodic movements in economic statistics, inasmuch as time is not a force. The causes of periodicity lie in the economic forces at work, but without knowing the forces—if sufficient periodicity exists—we may be able to derive the period. Men could predict the time of high and low tide from simple observations connecting the phenomena with the time of day centuries before they ever arrived at a theory which connected the tides with the attractive power of the moon, or even guessed at a dynamical theory of the tides.

In the following sections, an attempt has been made to derive the periods of the reserves and loans from the statistics, and then to consider, though meagrely, the causes at work producing the periodicity.

§ 30. Before commencing a study of the statistics of the reserves, a word should be said in regard to the *period-year*.

The period-year is the ideal numerical year. The first week ends on January 7. Since there are fifty-two weeks in the year plus a fraction of a week, the weekly statements published on Saturday do not fall for the first week on January 7 every year. It becomes necessary, therefore, to define as the first week, the weeks ending January 4, 5, 6, 7, 8, 9 and 10. The records are thus evenly divided. This method is used throughout the following pages.

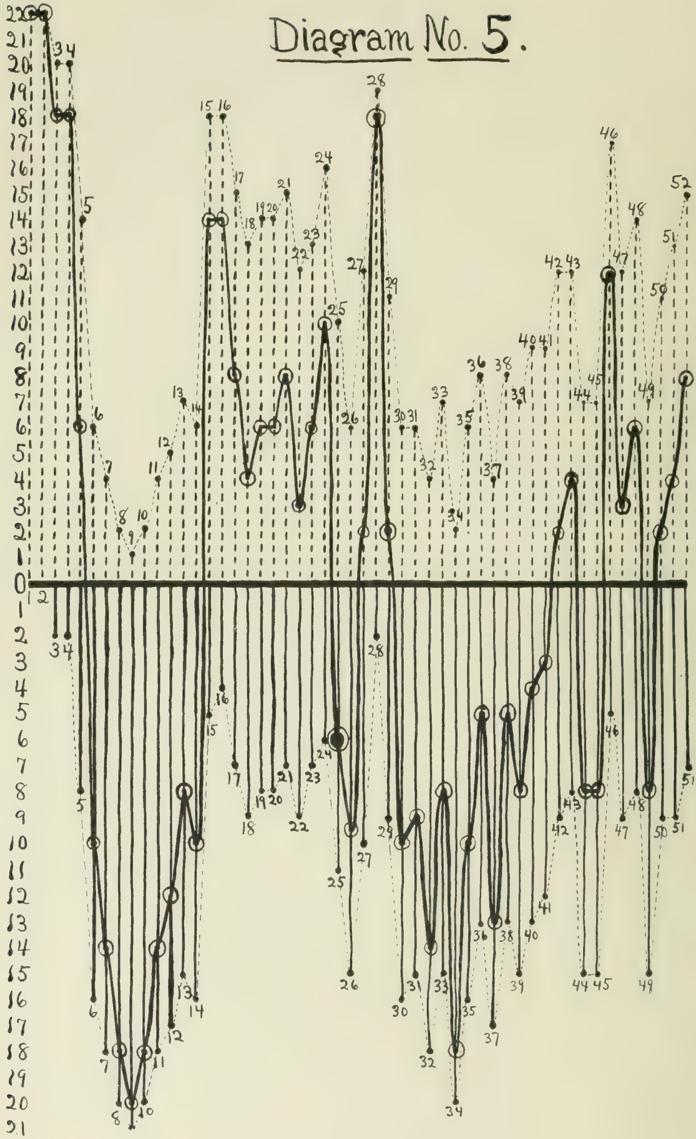
§ 31. From a table of the weekly changes in the reserve deviations (described in § 27), Table No. 10 has been compiled. In column I, are the numbers of years out of the

TABLE NO. 10.

Numerical Week.	I. Increases.	I. Decreases.	II. Excess Years.	III.	IV.	V.	Numerical Week.	I. Increases.	I. Decreases.	II. Excess Years.	III.	IV.	V.
1	22	0	+22	+1.00	+1.06	—0.10	27	12	10	+ 2	+ .09	+ .10	+1.56
2	22	0	+22	+1.00	+1.06	+0.96	28	19	2	+18	+ .78	+ .83	+2.39
3	20	2	+18	+ .82	+ .87	+1.83	29	11	9	+ 2	+ .09	+ .10	+2.49
4	20	2	+18	+ .82	+ .87	+2.70	30	6	16	—10	— .45	— .42	+2.07
5	14	8	+ 6	+ .27	+ .29	+2.99	31	6	15	— 9	— .41	— .39	+1.68
6	6	16	—10	— .45	— .42	+2.57	32	4	18	—14	— .64	— .60	+1.08
7	4	18	—14	— .64	— .60	+1.97	33	7	15	— 8	— .36	— .34	+0.78
8	2	20	—18	— .82	— .77	+1.20	34	2	20	—18	— .82	— .77	+0.01
9	1	21	—20	— .91	— .85	+0.35	35	6	16	—10	— .45	— .42	—0.41
10	2	20	—18	— .82	— .77	—0.42	36	8	13	— 5	— .23	— .22	—0.63
11	4	18	—14	— .64	— .60	—1.02	37	4	17	—13	— .59	— .56	—1.19
12	5	17	—12	— .09	— .08	—1.10	38	8	13	— 5	— .23	— .22	—1.41
13	7	15	— 8	— .36	— .34	—1.44	39	7	15	— 8	— .36	— .34	—1.75
14	6	16	—10	— .45	— .42	—1.86	40	9	13	— 4	— .18	— .17	—1.92
15	18	4	+14	+ .64	+ .68	—1.18	41	9	12	— 3	— .14	— .13	—2.15
16	18	4	+14	+ .64	+ .64	—0.50	42	12	9	+ 2	+ .14	+ .15	—2.00
17	15	7	+ 8	+ .36	+ .38	—0.12	43	12	8	+ 4	+ .18	+ .19	—1.79
18	13	9	+ 4	+ .18	+ .19	+0.07	44	7	15	— 8	— .36	— .34	—2.13
19	14	8	+ 6	+ .27	+ .29	+0.42	45	7	15	— 8	— .36	— .34	—2.47
20	14	8	+ 6	+ .27	+ .29	+0.65	46	17	5	+12	+ .55	+ .58	—1.89
21	15	7	+ 8	— .36	— .38	+1.03	47	12	9	+ 3	+ .14	+ .15	—2.00
22	12	9	+ 3	+ .09	+ .10	+1.13	48	14	8	+ 6	+ .27	+ .29	—1.85
23	13	7	+ 6	+ .27	+ .29	+1.42	49	7	15	— 8	— .36	— .34	—1.79
24	16	6	+10	+ .45	+ .48	+1.90	50	11	9	+ 2	+ .09	+ .10	—1.69
25	10	11	— 6	— .05	— .05	+1.85	51	13	9	+ 4	+ .18	+ .19	—1.50
26	6	15	— 9	— .41	— .39	+1.46	52	15	7	+ 8	+ .36	+ .38	—1.12

twenty-two years of our study, which have shown for each week of the period-year increases and decreases in the reserve deviations. Thus during the first week every year has shown

Diagram No. 5.



an increase. In the fifth week, increases have occurred in fourteen years and decreases in eight years. Column I is represented in diagram No. 5.

Diagram No. 5 is a "suggestion" picture of the weekly changes in the reserves during twenty-two years. The length of the dotted line above the horizontal axis measures for that week of the period-year the number of years out of the twenty-two years which have shown *increases* in the reserve deviations. The heavy black lines in the same way denote the number of years which have shown *decreases*. By joining the upper extremities of the dotted lines and the lower extremities of the black lines, we plainly distinguish a band-like movement which is worthy of further investigation.

A cursory examination shows heavy increases during January, heavy decreases during February and March, a majority of increases for the months of April, May, June and July, with the exception of a sharp decrease for the weeks containing the dates of July 1 and July 4. By August, the chart shows decreases in the majority and this tendency continues until late into the fall. By December 1, however, increases again predominate.

To illustrate this movement in another way, column II (Table No. 10) is formed by subtracting for each week the decreases from the increases and in prefixing a negative sign when the decreases exceed the increases. Column II is graphically represented in Diagram No. 5 by the circled dots. The circled dots record the excess number of years showing increases or decreases for each week of the period-year. By joining the dots the same periodic movement may be discerned.

§ 32. Although this diagram shows the weekly changes in the fund, it does not show the continuing movement of the fund itself throughout the year. To get an idea of this period with a minimum of arithmetical work, the following method was devised:—

(1) Divide through the excess number of years showing increases or decreases for each week (column II) by the total

number of years (in this case 22). This operation results in a percentage table (column III).

(2) Find in this percentage table (column III) the sum of the positive quantities and the sum of the negative quantities. Subtract the latter from the former and distribute this difference over the fifty-two records by subtracting algebraically the quotient of this difference divided by the sum of the two sums for each unit of advance, and adding the same quotient for each unit of decline. In this corrected table, the sum of the advances equals the sum of the declines. Thus the difference of the two sums is 1.29. The quotient 1.29/21.87 is equivalent to 0.059 for each unit of fluctuation. The corrected figures, obtained by adding the correction (0.06 × each unit of advance) to each advance, and subtracting a like product from each decline without regard to signs, are in column IV.

(3) Add the record for each consequent week to the algebraic sum of all the antecedent weeks, counting advances positive and declines negative. This results in a table of the relative position of the fund at any week with reference to all other weeks.

(4) To balance the curve on a horizontal axis, find the average of all the records. By subtracting this average from each record, the table of the annual relative period is obtained. Thus the successive algebraic sums are found from column IV and the average of these sums, 1.16, is subtracted from each of the fifty-two records. The resulting period (column V) is represented by the dotted line in Diagram No. 8.

A glance at the movement of the fund displayed by the dotted line suggests a well defined period in the total reserves. It is not a quantitative period, for the amount of deviation does not enter. It is rather a period of the degree of occurrence of the position of the fund with the week of the period-year. The value of this method lies in its arithmetical convenience in tentative work. It is useful as a prospecting method. The suggestion is, however, one of degree. A quantitative expression is required.

§ 33. Quantitative indices may be obtained by averaging for twenty-two years the weekly changes of the reserve deviations for each week of the period-year. These averages of the weekly changes may be consulted in Table No. 11 (22 year column). As the sign is positive or negative, the change is an increase or a decrease.

TABLE NO. 11.

Average Changes in the Reserve Deviations for each week of the Period-Year, with Probable Errors.

Week.	22 years.	18 years.	Week.	22 years.	18 years.	Week.	22 years.	18 years.
1	+4.686±.3	+4.783±.4	19	+2.068±.5	+1.994±.5	36	—1.118±.4	—1.639±.4
2	+5.323±.3	+5.617±.3	20	+1.018±.5	+1.778±.5	37	—1.304±.4	—1.322±.3
3	+3.105±.3	+3.428±.4	21	—0.023±.7	+1.100±.3	38	—1.068±.5	—0.561±.6
4	+3.018±.3	+2.533±.3	22	+0.723±.6	+0.400±.5	39	—0.541±.6	—0.706±.7
5	+0.082±.5	—0.628±.4	23	—0.373±.9	+1.278±.3	40	—0.327±.4	—0.261±.4
6	—2.300±.8	—0.805±.4	24	+1.623±.5	+1.550±.4	41	—0.359±.4	—0.644±.5
7	—2.141±.5	—1.311±.5	25	—0.136±.3	+0.156±.3	42	+0.800±.5	+0.461±.6
8	—2.986±.3	—3.105±.4	26	—0.759±.4	—0.933±.4	43	+0.800±.4	+0.761±.5
9	—2.919±.3	—2.818±.3	27	+0.245±.4	+0.333±.5	44	—1.100±.3	—1.611±.4
10	—2.374±.3	—2.322±.4	28	+2.241±.4	+2.422±.5	45	—0.823±.3	—0.611±.4
11	—1.136±.5	—1.606±.4	29	+0.018±.3	—0.094±.4	46	+2.613±.5	+1.622±.3
12	—0.836±.3	—1.000±.4	30	—0.714±.4	—0.878±.3	47	+0.886±.4	+0.133±.4
13	—0.723±.3	—0.723±.3	31	—1.518±.4	—1.622±.4	48	—0.323±.5	—0.089±.4
14	—1.323±.4	—1.027±.5	32	—2.364±.4	—2.133±.4	49	—0.891±.5	—0.144±.4
15	+1.823±.4	+3.144±.4	33	—2.050±.5	—2.061±.5	50	+0.359±.3	+0.711±.3
16	+2.218±.4	+2.083±.4	34	—1.673±.3	—2.056±.2	51	+0.313±.3	+0.678±.3
17	+1.091±.3	+0.822±.3	35	—1.555±.4	—2.133±.5	52	+0.500±.3	+0.511±.4
18	+0.809±.5	+0.539±.6						

Diagram No. 6 affords a graphic representation of the average changes in the reserve deviations for each week of the period-year on the twenty-two year basis. The length of the vertical, dotted line measures the average change for that particular week, and as the line extends upwards or downwards the change is an increase or a decrease. The communities of changes all in one direction or the other at different seasons of the year are significant.

Before going into details, however, it may be well to inquire whether these averages are the most representative that can be derived from this statistical array. It is possible that extraordinary events have occurred in some years, so extraordinary as to quite distort the average or the normal. The Chart showing the actual course of the reserve devia-

tions during the twenty-two years, will be remembered. During 1893 occurred a crisis which in its severity was unequalled in all the last quarter of the century. The year 1894 was full of governmental bond operations and the after-effects of the crisis. So extreme were the deviations from the axis that more or less distortion must necessarily result if such years are included in the averages.

To remedy this distortion, the averages in the second column, Table No. 11, were calculated. The sole difference between the two series is the omission of the years 1879, 1893, 1894 and 1900 from the latter. The eighteen year averages are represented in Diagram No. 6 by the heavy vertical lines.

On the whole, the differences between the two series are not striking. How are we to decide which of the two sets of averages is more representative of the normal movement? This question we can settle without being obliged to resort to general impressions. It is only necessary to calculate the probable variations of the averages in the two series, compare the two sets of probable variations, and to select as the more representative the series having the smaller probable variation.

The method of calculating the probable variation is doubtless familiar. The expression of the probable variation is approximately $\pm 2/3\sqrt{\frac{\Sigma v^2}{n-1}}$, where v is the deviation of any one of the numbers entering in to form the average from the average, n the number of records forming the average.*

* (v)	(v^2)
−0.6	0.36
+1.2	1.44
+2.1	4.41
−0.2	0.04
+3.7	13.69
−0.6	0.36
+3.7	13.69
−0.1	0.01
−1.0	1.00
−0.6	0.36
....	....
−2.3	5.29
	$\Sigma v^2 = 54.83$

To illustrate briefly, the average change for the second week in the eighteen year series is +5.617. The numbers entering the average are eighteen in number. The differences between the successive numbers and the average are given in the column under v. The squares of the v's follow and on adding the column we have 54.83, which is the Σv^2 of the expression $\pm 2/3\sqrt{\frac{\Sigma v^2}{n-1}}$. $\therefore 54.83 \div 17 = 3.22$. The square root of 3.22 multiplied by 2/3 is 1.2 or the probable variation of the statistical array.

Diagram No. 6.

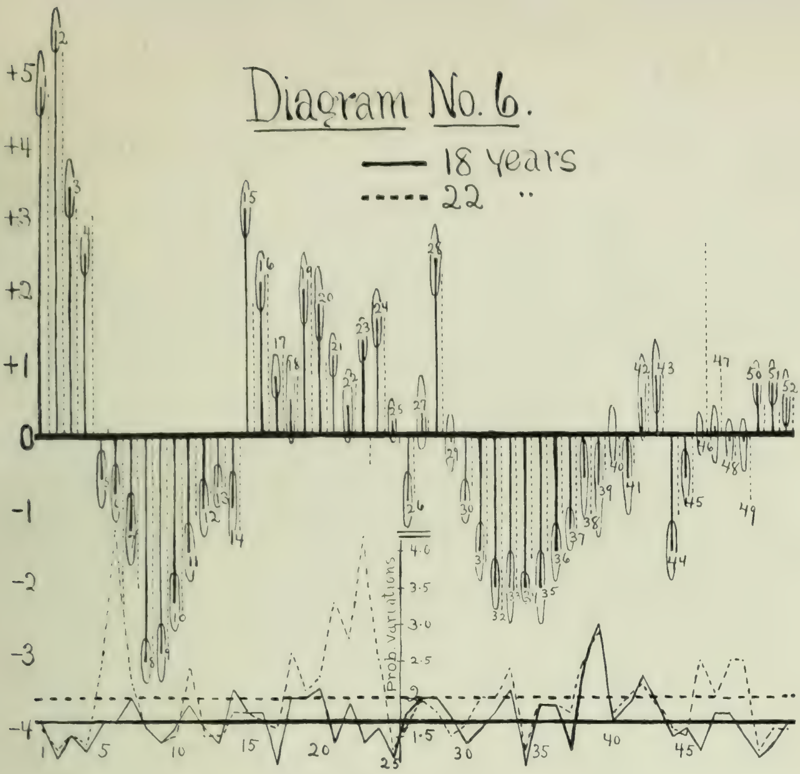


Diagram No. 7.

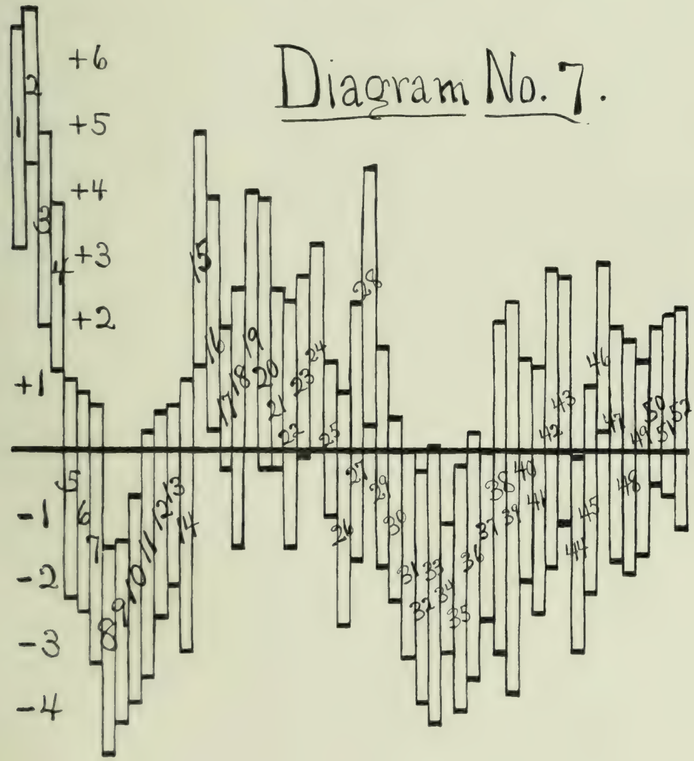


Table No. 12 contains the two series of probable variations. In Diagram No. 6 the broken, dotted line (in the lower part) represents the twenty-two year series of probable variations, and the broken, heavy line the eighteen year series. The

TABLE NO. 12.

Probable Variations of the eighteen year and twenty-two year Average Changes in the Reserve Deviations.

Week.	18 years.	22 years.	Week.	18 years.	22 years.	Week.	18 years.	22 years.	Week.	18 years.	22 years.
1	1.7	1.6	14	2.1	1.8	27	2.0	2.0	40	1.7	1.8
2	1.2	1.3	15	1.8	1.8	28	2.0	1.8	41	1.9	2.0
3	1.5	1.5	16	1.8	1.7	29	1.7	1.5	42	2.3	2.2
4	1.3	1.4	17	1.1	1.6	30	1.4	1.7	43	1.9	1.9
5	1.7	2.5	18	2.0	2.6	31	1.6	2.0	44	1.5	1.6
6	1.7	4.3	19	2.0	2.1	32	1.8	2.0	45	1.6	1.5
7	2.0	2.2	20	2.1	2.3	33	2.1	2.4	46	1.3	2.5
8	1.6	1.6	21	1.4	3.3	34	1.0	1.3	47	1.8	2.0
9	1.4	1.4	22	1.9	2.8	35	1.9	1.9	48	1.8	2.5
10	1.6	1.5	23	1.4	4.2	36	1.9	1.9	49	1.5	2.5
11	1.9	2.4	24	1.6	2.5	37	1.3	1.8	50	1.2	1.3
12	1.6	1.5	25	1.2	1.3	38	2.5	2.5	51	1.4	1.6
13	1.4	1.5	26	1.8	1.7	39	3.0	2.9	52	1.7	1.6

straight, dotted line is the average of the probable variations for the fifty-two weeks of the former series and the straight, heavy line the average for the latter series. The numerical values are 1.7 and 2.0 respectively. There is thus a 0.3 difference in favor of the eighteen year averages, and consequently all further investigations will be connected solely with the more representative eighteen year series.

In passing, it may be said that the movement of the broken heavy line is of some significance in showing at what times of the year the forces producing the movement are most uncertain. For instance, the thirty-eighth and thirty-ninth weeks, which are the last two weeks of September, have a wide variation from the yearly average variation, with the actual average of the changes slightly negative. The meaning evidently is that there is a great difference in different years during these weeks with respect to the continuance of the outward movement of currency. In some years it continues into October and in other years ceases by the second

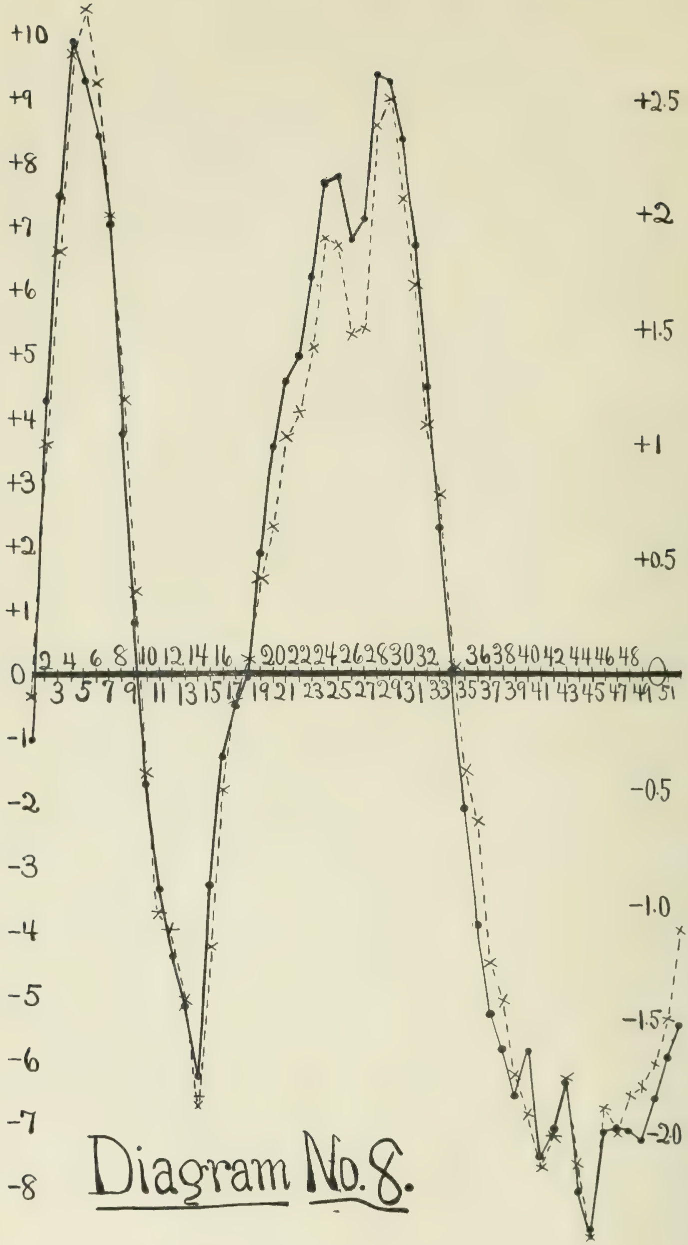
week in September. This line is then the gauge of the internal uncertainty of the different years with regard to the action of these forces. On the other hand, a week like the thirty-fourth, which ends on August 26th, has a probable variation of 1.0, with a negative average change of 2.2. The probability of the outward movement for this week is thus very great and is in anticipation of the needs of depository banks for the first of September uses.

Knowing the probable variations from Table No. 12, the probable errors of the averages are easily found. For the probable error of the average or mean is simply $1\sqrt{n}$ of the probable variation. In Diagram No. 6, the ovals at the extremities of the vertical lines signify the probable errors to which the eighteen year averages are subject.

§ 34. The ranges of probable variations are expressed graphically in Diagram No. 7. The rectangles are the boxes within which the chances are even that the changes will lie for any week of any year. Thus a man betting the same amount at even odds each week over eighteen years, according to this "system" should theoretically end the eighteenth year without loss or gain. These boxes represent the actuarial range.

It is perhaps some approximation to exactness when for eighteen years an underwriter would have been safe in betting that there would be an increase in the second week of the year between 4.4 and 6.8, or for the eighth week a decrease between 1.5 and 4.5. Suppose we take the second week of the year 1901 and estimate the limits of the probable range of increase. The interpolated axis for the second week is 217.5 by the equation $\text{Log } R = 1.823682 + 0.023317\ t$. The increase should then lie with an even chance between 4.4×217.5 and 6.8×217.5, i. e. between an increase of \$9.5 millions and \$14.8 millions. The actual increase for that week was \$11.8 millions approximately, or 5.4% change with the eighteen year average 5.3%. Of course such close prediction is by no means always possible.

On the other hand, a deviation from the range seldom



+10
+9
+8
+7
+6
+5
+4
+3
+2
+1
0
-1
-2
-3
-4
-5
-6
-7
-8
+2.5
+2
+1.5
+1
+0.5
-0.5
-1.0
-1.5
-2.0
2 4 6 8 10 12 14 16 20 22 24 26 28 30 32 36 38 40 42 44 46 48
3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51

Diagram No. 8.

occurs without the bias extending over several weeks and often over the whole season. Thus the range is capable of correction by the dynamic tendencies of the immediately preceding weeks.*

The movement with the seasons of the year is apparent by the band movement of the rectangles.

§ 35. To return to the table of the eighteen year series, a slight correction is necessary before proceeding further. If we add the advances in one column and the declines in another, the sum of the advances, 38.837, is greater than the sum of the declines, 34.843, by 3.994. This disparity occurs for several reasons,—among which are imperfect interpolation, a majority of the years recently above the axis and the omission of the years 1893 and 1894.

To form the period curve it is necessary that the sum ot the advances shall equal the sum of the declines. This is readily accomplished by a distribution of the 3.994 difference over the 73.670 points of movement by deducting (3.994/73.670) or 0.054 from each unit of advance and adding 0.054 to each unit of decline. This correction does not apprecia-bly affect the yearly movement. The corrected averages are given in Table No. 13.

TABLE NO. 13.

Average Weekly Changes (corrected) and Annual Period of the Reserve Deviations.

Week.	Corrected Changes.	Reserve Period.	Week.	Corrected Changes.	Reserve Period.	Week.	Corrected Changes.	Reserve Period.	Week.	Corrected Changes.	Reserve Period.
1	+4.524	—1.037	14	—1.081	—6.246	27	+0.315	+7.135	40	—0.275	—5.868
2	+5.315	+4.278	15	+2.977	—3.269	28	+2.292	+9.427	41	—0.679	—7.547
3	+3.244	+7.522	16	+1.975	—1.294	29	—0.099	+9.328	42	+0.436	—7.111
4	+2.398	+9.920	17	+0.779	—0.514	30	—0.926	+8.402	43	+0.720	—6.391
5	—0.662	+9.258	18	+0.512	—0.003	31	—1.708	+6.694	44	—1.697	—8.088
6	—0.842	+8.410	19	+1.886	+1.883	32	—2.246	+4.448	45	+0.643	—8.731
7	—1.381	+7.029	20	+1.681	+3.564	33	—2.174	+2.274	46	+1.536	—7.195
8	—3.272	+3.757	21	+1.041	+4.605	34	—2.169	+0.005	47	+0.128	—7.067
9	—2.969	+0.788	22	+0.378	+4.983	35	—2.246	—2.141	48	—0.090	—7.157
10	—2.446	—1.658	23	+1.208	+6.191	36	—1.725	—3.868	49	—0.152	—7.309
11	—1.629	—3.350	24	+1.464	+7.655	37	—1.392	—5.258	50	+0.673	—6.636
12	—1.054	—4.404	25	+0.174	+7.802	38	—0.591	—5.849	51	+0.642	—5.994
13	—0.761	—5.165	26	—0.982	+6.820	39	—0.744	—6.593	52	+0.483	—5.531

* This method of correction is reserved for future discussion.

If now we plot each of the corrected changes, not from a horizontal axis as in Diagram No. 6, but from the level at which the preceding weekly change ends, and then join the extremities of the lines which are farther from the levels, the connecting line shows both the increase for the week and the position of the fund in any week with reference to all other weeks.

It is a simple task to construct this table. The advance for the second week is added to the advance for the first week. All succeeding advances are added to the preceding sums, and all succeeding declines are subtracted. The successive records are added and the sum is divided by 52. This average of the period (5.561) is the axis upon which the period curve is balanced by subtracting the average from each one of the weekly records.

The *annual period of the reserves* (Table No. 13, second column) is represented in Diagram No. 8 by the heavy line. The dotted line is the "degree of occurrence" period already noted in § 32. The value of this "occurrence" method* appears in its prospective accuracy (shown by the close correspondence between the two periods) as well as in its ease of calculation.†

§ 36. A glance at Diagram No. 8 discloses two movements of currency into the banks and two movements out. One movement commences about the forty-fifth week, November 11th, and culminates in the fourth week of the year, the week ending January 28th, after an advance of nearly nineteen points.

The decline beginning with the fifth week of the year, the week ending February 4th, ends about the fourteenth week,

* Since the above was written, there has been published a paper by Yule, *On the Association of Attributes in Statistics* (Phil. Trans., Vol. 194, London, 1900), which covers what I have called here "degree of occurrence" with great fullness.

† How the quantitative period may be obtained directly from the 'occurrence' period by a 'scale coefficient' without calculating the averages, will be discussed elsewhere. It is sufficient, in this place, to point out the considerable saving of computation.

or April 8th, after a continuous decline of about sixteen points.

The advance commences at once. The culmination of the advance after one recession comes about the twenty-eighth week, the week ending July 15th, after an advance of fifteen and one-half points.

The ensuing decline, beginning about August 1st, continues with two recessions to the low point of the year in the forty-fifth week, the week ending November 11th, after the largest decline of the year, 18 points. With a growth axis value of $220 millions, this means a movement of nearly one-fifth or approximately forty millions of dollars. Bankers have long been aware of* and, in fact, yearly reckon upon this movement. This is the first attempt, perhaps, to reduce this movement to a quantitative expression.

§ 37. A number of more or less interesting topics arise. Among these, the most important is the meaning of this periodicity.

The primary cause of these two movements is unquestionably the temperature. The temperature in its variations makes the seasons of the year and so controls the activities of production. The rapid decline of sixteen points during the last month of winter and the first month of spring is partly due to the outflow of currency to the out-of-town and western banks. Loans expand and more money is needed by these banks for reserves and more money is needed for pocket use by the people. Why? Because the farmer is getting ready to plant his crops. He has to buy fertilizers and seed. If he buys by cash, more money is needed for exchange; if by credit, more money is needed for reserve for the loans which somewhere must exist. The decline then anticipates the sowing in all its branches.

By the middle of March the need begins to pall, and,

* Professor Sumner, writing in 1876 of the currency between 1850–60, says: "The same course of events, more or less marked, occurred throughout this period. Currency flowed to New York during the summer, was loaned on call (interest being paid for deposits), was withdrawn in the fall, producing contraction of loans and stringency." (*History of American Currency*, pp. 176–7.)

gradually, as loans are paid and commodities bought, the money flows back to New York. The crops are growing over the land. The farmer has bought his supplies. The spring and summer clothes have been procured. It is the early summer and mid-summer inactivity. Business is dull.

There is one break in this slackening need for exchange, and this occurs in the weeks ending July 1st and 8th. Two big events occur in these weeks. The mid-year settling date is followed within three days by Fourth of July. The heavy payments in dividends have a temporary effect. Wages are often prepaid over the holiday. Families go off for the day and pocket money is needed. Thus the holiday expenditure is reflected in the second retreating dot. So the pleasures as well as the labors of a people may be read from the statistical records of the past.

The next two weeks in July show the maximum summer funds. By the last week of July the tide turns. It is in expectancy of the harvest. Dry goods houses begin to lay in their fall stocks and expand their credits. The farmer's wife plans her fall hat and winter garments against the ripening crops. The harvest furnishes the means to sink the debit in the loans. More pocket money is again needed. There is a slight recession after the settling day of October 1st, another recession after the 15th.

The great transactions of the harvests, the moving of the crops well under way, a slackening comes in the long decline, which by the end of the first week of November is turned into the mid-winter advance. During December and January the cycle repeats itself. The reserves of the banks again rapidly rise.

§ 38. A more technical investigation of the annual period of the reserves is now under way, but the statistics necessary for results are so widely scattered that I can do little more than outline the main avenues of research.

There are four great classes of institutions which are constantly taking from and adding to the reserve fund of the banks. These are, (1) The United States Treasury, (2) The

Out-of-town Depository Banks, (3) The Local Business Houses and Trust Companies, (4) The Foreign Exchange Dealers. The annual period, which we have found, represents the average net results of the operations of these four great classes of institutions.

At this time it may be well to consider topically each one of the four classes in order.

(1) *The United States Treasury* is all the time affecting and must forever affect the money market by its daily great transactions. In some months of the year, its expenditures are greatly in excess of its receipts. At other times, the opposite is the case. Thus in the money market, December is considered an unfavorable month so far as governmental operations are concerned, inasmuch as the expenditures are small owing to no interest payments and lighter pension disbursements. The result is a hoarding of currency by the government. In March, too, there are no interest payments and, at the same time, the receipts are generally the largest of the year.

This locking up of currency at times when business is in need of every dollar that exists, or again, the opening of the flood gates of the United States Treasury at times when business does not stand in need of surplus cash, in no way regulated by sound finance, but dependent on the many chances that operate upon the incoming revenues and outgoing expenditures of the Government, has long been a source of vexatious annoyance to the financial world.

The present temporary remedy is for the Secretary of the Treasury to deposit the incoming revenues with the banks and to buy bonds at a premium, thus releasing the surplus in the Treasury. When a sound, permanent method* of relief is adopted, it is probable that a National Bank of the United States will figure prominently among the details.

* Since the above was written, the plan for a "central bank, with a head in the chief commercial city, with branches in each of the commercial centers, which shall constitute the head and the backbone of the system," has been strongly advocated by Mr. A. B. Stickney, President of the Chicago Great Western Railway Company, before the 27th Annual Convention of the American Bankers Association.

(2) The *Out-of-town Banks* find this advantage in depositing surplus cash in the New York banks. The New York banks allow them a small rate of interest, about 2%, and these deposits the out-of-town banks may count to a certain proportion (§ 5) as reserve.

During the seasons of the year in which business demands are less outside of New York, the out-of-town deposits rapidly swell, as we have already noted. The outflow* from New York in the last half of the year has generally two main movements. The first is to the South to move the cotton crop and culminates in October. The second outflow, to the West in connection with the harvests, ends in November or December. There is in addition a third minor outflow to the far South at its maximum about the middle of November in connection with the movement of the sugar crop. I mention these in detail to show the complexity of the influences at work, which after all in their simplicity are but the manifestations of a changing temperature,† and as with the passage of years we come to think of a crop period—of the sowing and of the garnering of the harvest, which in the long average is an assured fact, so in the bank periods, the needs of to-day were primarily the needs of the years passed. For we still gather our food from the ground, and prosperity or depression finds its ultimate root in plenty or scarcity of crops.

Say what we may of the growing importance of manufactures, it is still mainly local differences that we are considering. Let there be for three years in succession a

* The banks in the West and South obtain during the time of harvesting and moving the crops very high rates at home. At times, the demand for loans is so great during this period that the banks pay as high as five or six per cent. to depositors for balances.

† To show the correlation between the annual period of the temperature and the reserves and loans, a glance at Diagram No. 18 will suffice. The line connecting the circled dots represents the average annual period of the temperature calculated from the ten-year averages of New York City, Chicago, St. Louis, New Orleans and Denver.

Month,	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Average,	35	37	43	53	64	72	76	78	67	57	44	36

destitution of all ground crops. Then our populations on the whole go starving, and the wheels of industry cease to turn. For what staple of agriculture can show anywhere near a constant stock three fold the annual consumption?

It is like an island nation whose life supplies come from beyond its confines. Girt England for a year with a naval cordon, artificial as a ship blockade may be. Grant that no food supplies shall pass the cordon for that length of time. Her six weeks stocks would turn the hum of wheels from the weird noises of a mad-house into the dumbness of a charnel factory.

Laws may change, theories may change, local changes of vast importance may come to pass, but the primary facts on which this period rests are not ruled by laws of men, nor modified by theories of reformers, nor escaped by local shifting. It rests on the facts of nature, on the temperature of hemispheres, on the very twistings of the earth upon its axis, and as long as summer follows winter and we obtain our foods from the soil of the earth by the growth processes of nature, somewhere ceaselessly responding will be found this banking rhythm.* The period may differ in different parts of the world to fit the times of sowing and of reaping, but the period is there implanted by the facts.

In any exchange system these facts must be forever met. To provide the currency for the farmer, greater elasticity is necessary. To provide for safety of values, elasticity must be limited by security. How are these facts best met—by small independent banks, by branch banks, or by some regulated output of currency when the seasons come around?

One necessity stands paramount. The banking facilities must be in a place where the farmer can get at them. To provide currency to fit the seasons requires more judgment than is in men. Indeed this remedy is held only by the dreamers. An automatic system depending on the caprice

* In 1853, "the currency set towards the financial centers, country banks keeping their balances generally in New York. These balances were required in the fall, and the withdrawal of them produced contraction and stringency at that season." Sumner, *History of American Currency*, pp. 175–6.

of no man has been found in the affairs of men to be, on the whole, the best. The practical question really lies between small independent banks* and branch banking. The discussion of this practical question will be deferred until that time when this whole subject can be statistically treated.

(3) The *Trust Companies* are doing in addition to their regular lines much the same business as the Associated Banks. The trust companies, however, carry very small reserves. It is their practice when money rates are low to allow the banks to loan out their balances. Higher money rates, however, tend to make trust companies withdraw their deposits from the banks and lend directly, "partly for advertising value, and partly to get the best rates obtainable."†

This factor then depends on the discount rate and produces a decline in banking reserves at a time when such a decline is viewed with apprehension. At the same time, the effect is more apparent than real, for the money is reloaned by the trust companies and in no way passes out of the local market. These operations are known only by hearsay to the economist. There are no satisfactory statistics with respect to the importance of this movement.

The same lack of accurate statistics exists with respect to the business houses and there is consequently little to be said.

(4) Next in importance to the interior movement of money is the foreign movement. In the *foreign exchange* market is a wealth of statistics very rare in economic science.

A study of the foreign exchange movements involves the discount rates of the principal banking centers of the world as well as the preparation of a carefully planned mathematical theory, the beginnings of which may be found in the chapter

* The success or lack of success of the present system of small independent banks, recently adopted, will be empirically determined during the years of liquidation which must sooner or later follow the present tendencies towards inflation. In the South and West the custom of lending on real estate security *indirectly* is prevalent. The security is the personal note of the borrower backed by a mortgage condition. In case the borrower is unable to pay, some officer of the bank pays off the loan and takes the real estate. In this way, the real estate escapes the bank's return to the Comptroller. As long as land values rise, this is profitable both to bank and officer.

† Wall Street Journal, October 9, 1900.

on *Exchange* by *Cournot*.* The statistical treatment of this subject will therefore be deferred until a larger amount of statistical material can be collected.

§ 39. Although a study of *pool activity* in the flotation of new securities belongs strictly under the dynamic element, there is an important connection at this point. The link that joins the subject of pools to periodicity in the reserves is the call discount rate. The rapidly increasing reserves at certain seasons of the year result in falling discount rates. The low discount rate enables the pools to borrow an abundance of capital with which to advance the quotations of selected securities. So the declining reserves in the fall and spring and the resulting rising discounts furnish the material of a campaign for lower prices by the pools. This is one reason why the New York banks are willing to pay 2% interest for the balances of out-of-town banks. New York, as the financial center of one hemisphere (if not of two), finds a use for the idle currency of the nation among the wholesale dealers of investments in their peculiar functions as well as among the smaller retail houses. The whole subject of pools is clouded in mystery. The book explaining the economic aspects, functions and methods of the pools or underwriting syndicates, with copious illustrations and statistical tables, has, unfortunately, yet to be written.

* Augustin Cournot, *Researches into the Mathematical Principles of the Theory of Wealth* (1838), translated by Nathaniel T. Bacon, with a bibliography of mathematical economics by Irving Fisher, 1897.

CHAPTER V

PERIODICITY IN THE LOAN DEVIATIONS

§ 40. The methods used in studying the periodicity of the reserve deviations may be applied to the loans. The years showing increases and decreases (Table No. 14) are repre-

TABLE NO. 14.

Number of Years showing Increases and Decreases in the Loan Deviations.

Week.	Increases.	Decreases.	Excess	Week.	Increases.	Decreases.	Excess.	Week.	Increases.	Decreases.	Excess.	Week.	Increases.	Decreases.	Excess.
1	14	5	+ 9	14	11	10	+ 1	27	13	8	+ 5	40	10	12	− 2
2	6	15	− 9	15	7	14	− 7	28	6	15	− 9	41	10	11	− 1
3	17	5	+12	16	8	14	− 6	29	5	14	− 9	42	4	17	−13
4	13	9	+ 4	17	9	12	− 3	30	7	13	− 6	43	7	15	− 8
5	18	2	+16	18	15	6	+ 9	31	17	4	+13	44	11	11	0
6	18	3	+15	19	10	11	− 1	32	16	5	+11	45	4	11	− 7
7	16	6	+10	20	9	11	− 2	33	13	9	+ 4	46	7	14	− 7
8	10	10	0	21	8	14	− 6	34	12	10	+ 2	47	10	12	− 2
9	15	5	+10	22	14	7	+ 7	35	6	16	−10	48	13	8	+ 5
10	11	10	+ 1	23	15	5	+10	36	13	9	+ 4	49	10	11	− 1
11	9	12	− 3	24	12	9	+ 3	37	9	13	− 4	50	8	14	− 6
12	6	16	−10	25	12	6	+ 6	38	5	15	−10	51	14	7	+ 7
13	5	15	−10	26	16	4	+12	39	5	16	−11	52	13	7	+ 6

sented in Diagram No. 9. The dotted lines above the horizontal axis measure the years with increases for each one of the fifty-two weeks, and the heavy lines the decreases.

Two movements are apparent:—(1) an annual movement resembling, in some degree, the period of the reserves; (2) an irregular monthly movement in which the rise falls on the week containing the first day of the month. This is shown by the saw-tooth, upper edge. The points of the teeth fall, as a rule, on the weeks containing the first days of the month. Just before the first days of the months, business men borrow money to meet obligations then coming due.

Diagram No. 9.

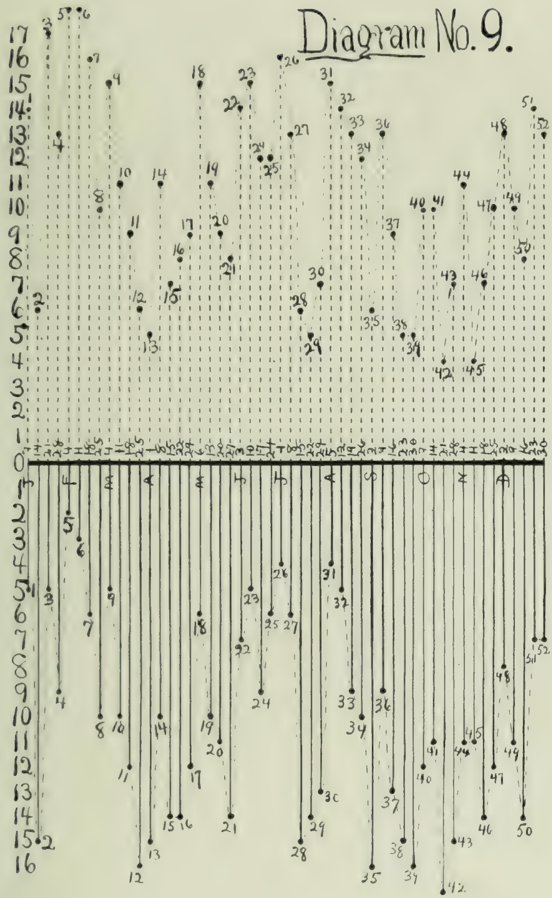


Diagram No. 10.

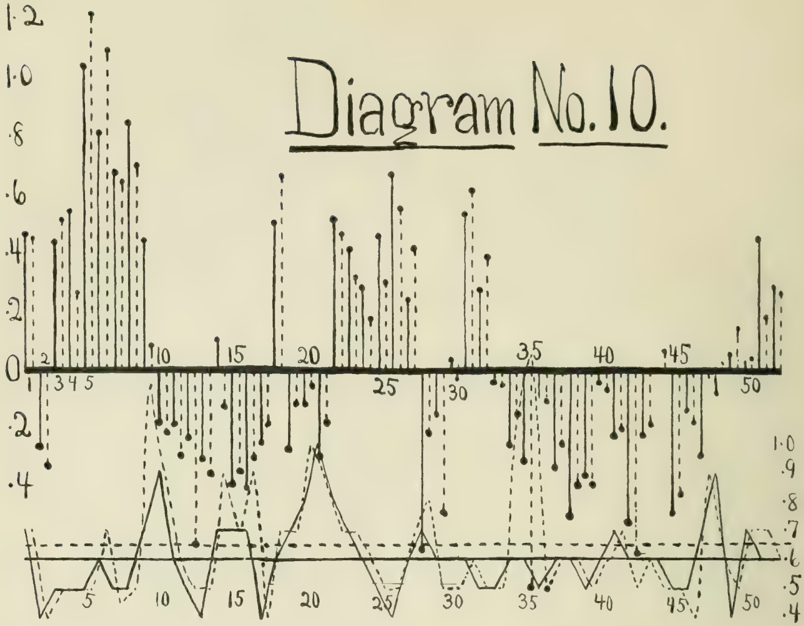


Diagram No. 11.

§ 41. The average weekly changes in the loan deviations, calculated both on the basis of twenty-two years and eighteen years, appear in Table No. 15. In the eighteen year averages, the years 1879, 1893, 1894 and 1900 are omitted, as in the reserve averages.

TABLE NO. 15.

Average Weekly Changes in the Loan Deviations on the basis of twenty-two years and eighteen years, with Probable Errors.

Week.	22 years.	18 years.	Week.	22 years.	18 years.	Week.	22 years.	18 years.
1	+0.445 ± 0.1	+0.467 ± 0.1	19	—0.109 ± 0.1	—0.267 ± 0.2	36	—0.105 ± 0.1	—0.094 ± 0.1
2	—0.327 ± 0.1	—0.261 ± 0.1	20	—0.045 ± 0.1	—0.106 ± 0.2	37	—0.250 ± 0.1	—0.328 ± 0.1
3	+0.509 ± 0.1	+0.438 ± 0.1	21	—0.177 ± 0.2	—0.294 ± 0.2	38	—0.382 ± 0.1	—0.500 ± 0.1
4	+0.264 ± 0.1	+0.536 ± 0.1	22	+0.459 ± 0.1	+0.511 ± 0.2	39	—0.391 ± 0.1	—0.359 ± 0.1
5	+1.208 ± 0.1	+1.033 ± 0.1	23	+0.364 ± 0.1	+0.411 ± 0.2	40	+0.055 ± 0.1	—0.045 ± 0.1
6	+1.082 ± 0.1	+0.800 ± 0.1	24	+0.173 ± 0.1	+0.278 ± 0.1	41	—0.191 ± 0.1	—0.211 ± 0.2
7	+0.632 ± 0.1	+0.616 ± 0.1	25	+0.300 ± 0.1	+0.411 ± 0.1	42	—0.632 ± 0.1	—0.523 ± 0.1
8	+0.068 ± 0.1	+0.083 ± 0.1	26	+0.555 ± 0.1	+0.672 ± 0.1	43	—0.173 ± 0.1	—0.222 ± 0.1
9	+0.077 ± 0.3	+0.435 ± 0.2	27	+0.414 ± 0.1	+0.241 ± 0.1	44	+0.064 ± 0.1	—0.000 ± 0.1
10	—0.218 ± 0.2	—0.178 ± 0.1	28	—0.213 ± 0.1	—0.611 ± 0.2	45	—0.468 ± 0.1	—0.639 ± 0.1
11	—0.291 ± 0.1	—0.178 ± 0.1	29	—0.477 ± 0.2	—0.150 ± 0.1	46	—0.177 ± 0.1	—0.136 ± 0.1
12	—0.595 ± 0.1	—0.237 ± 0.1	30	—0.018 ± 0.1	+0.033 ± 0.1	47	—0.018 ± 0.2	—0.295 ± 0.2
13	—0.364 ± 0.1	—0.306 ± 0.1	31	+0.514 ± 0.1	+0.483 ± 0.1	48	+0.027 ± 0.2	—0.073 ± 0.2
14	—0.118 ± 0.2	+0.100 ± 0.2	32	+0.386 ± 0.1	+0.267 ± 0.1	49	+0.141 ± 0.1	+0.050 ± 0.1
15	—0.350 ± 0.1	—0.383 ± 0.2	33	—0.045 ± 0.1	—0.033 ± 0.1	50	—0.045 ± 0.1	+0.032 ± 0.2
16	—0.291 ± 0.2	—0.400 ± 0.2	34	—0.136 ± 0.2	—0.250 ± 0.1	51	+0.186 ± 0.1	+0.432 ± 0.1
17	—0.173 ± 0.1	—0.237 ± 0.1	35	—0.736 ± 0.3	—0.311 ± 0.1	52	+0.259 ± 0.1	+0.288 ± 0.1
18	+0.664 ± 0.1	+0.500 ± 0.1						

The eighteen year and twenty-two year series are drawn in one Diagram (No. 10) for purposes of comparison. The heavy lines stand for the eighteen year averages and the dotted lines for the twenty-two year series.

Both sets of averages are subject to one well defined tendency. Increases are the order from the week ending January 21st to the week ending March 4th, decreases from March 11th to May 27th with the strong exception of the week containing May 1st. Increases then follow up to the end of the first week of July. Three weeks of decreases ensue followed by two weeks of increases for the first two weeks of August, and from the middle of August a general declining tendency continues to the end of November with slight increases for October 1st and November 1st. From

December 1st the loans increase with the exception of the recession for December 15th. This is the general movement of the year. It is interesting also to note how plainly the first days of each month produce their effects, often completely counter-balancing the prevailing tendency.

§ 42. The differences between the two sets of averages are not striking. In order to select the more representative series, the probable variations (Table No. 16) were calculated

TABLE NO. 16.

Probable Variations of the eighteen year and twenty-two year Average Changes in the Loan Deviations.

Week.	18 years.	22 years.	Week.	18 years.	22 years.	Week.	18 years.	22 years.	Week.	18 years.	22 years.
1	0.7	0.7	14	0.7	0.9	27	0.6	0.7	40	0.6	0.6
2	0.4	0.4	15	0.7	0.7	28	0.7	0.8	41	0.7	0.7
3	0.5	0.5	16	0.7	0.9	29	0.6	0.5	42	0.6	0.5
4	0.5	0.5	17	0.4	0.4	30	0.6	0.5	43	0.6	0.6
5	0.5	0.5	18	0.6	0.7	31	0.6	0.6	44	0.6	0.5
6	0.6	0.7	19	0.7	0.7	32	0.5	0.5	45	0.5	0.5
7	0.5	0.4	20	0.8	1.0	33	0.5	0.5	46	0.5	0.4
8	0.5	0.5	21	1.0	0.9	34	0.6	1.1	47	0.7	0.9
9	0.7	1.2	22	0.8	0.7	35	0.6	1.3	48	0.9	0.6
10	0.9	0.8	23	0.7	0.7	36	0.5	0.5	49	0.4	0.5
11	0.6	0.6	24	0.6	0.6	37	0.6	0.6	50	0.7	0.7
12	0.5	0.5	25	0.5	0.5	38	0.6	0.6	51	0.6	0.7
13	0.4	0.5	26	0.4	0.5	39	0.5	0.5	52	0.6	0.6

for each week, as in the case of the reserves. The dotted line in the lower half of Diagram No. 10 joins the ordinates of the twenty-two year variations, the heavy line the eighteen year series.

It is obvious that the heavy axis (the average of the eighteen year variations) is lower than the dotted axis (the average of the twenty-two year variations). It follows that the eighteen year changes are the more representative, on the average, by about 0.1.

At several points the broken heavy line advances above the axis. These oscillations show that the forces producing them vary widely between different years. They are connected with loans on foreign exchange, excess of exports or imports and, of course, with the cotton and grain movements.

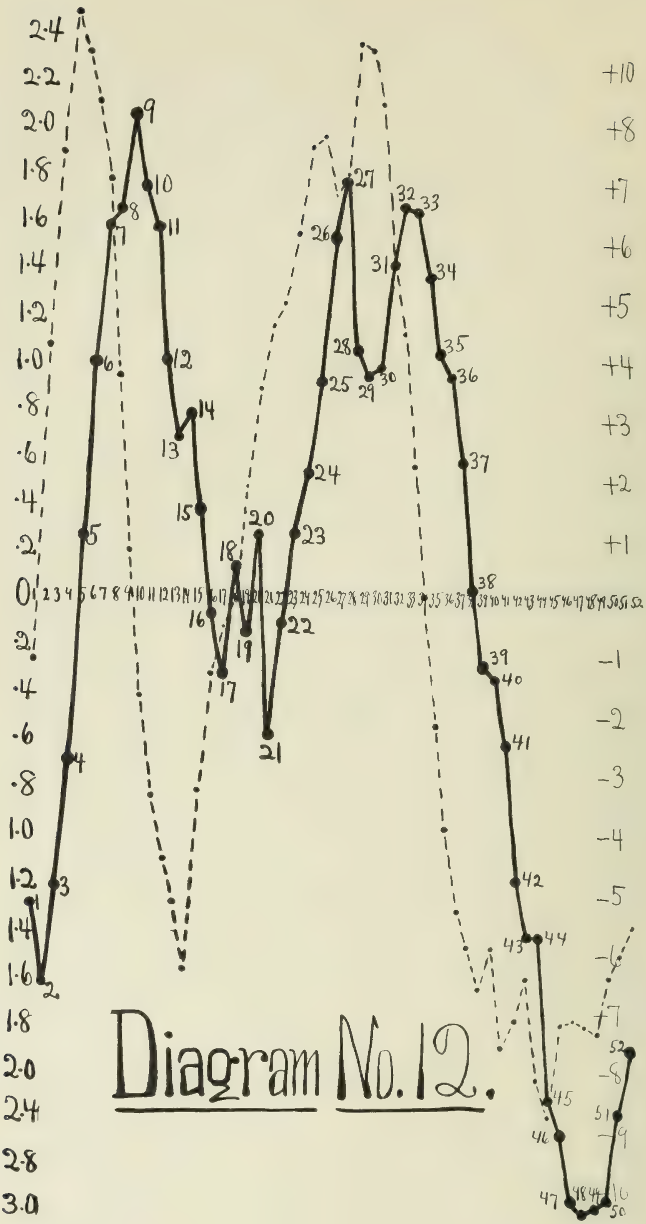


Diagram No. 12.

§ 43. The band movement of the probable variations of the loans is represented by Diagram No. 11. The rectangles are the limits within which the chances are even that the changes will lie for the various weeks. These ranges are percentagely much smaller than the ranges found for the reserves. In other words, the ratio of the amplitude of period to growth axis is less in loans than in reserves.

§ 44. Before constructing the *annual period* of the loans, it is necessary to correct the eighteen year averages by subtracting 0.074 from each unit of advance and adding 0.074 to each unit of decline. From the corrected averages (Table No. 17, first column), the loan period is obtained in exactly the same manner as in the reserves. The loan period (Table

TABLE NO. 17.

Annual Period of the Loans.

Week.	Corrected Averages.	Annual Period.	Week.	Corrected Averages.	Annual Period.	Week.	Corrected Averages.	Annual Period.	Week.	Corrected Averages.	Annual Period.
1	+0.432	—1.340	14	+0.093	+0.770	27	+0.223	+1.757	40	—0.049	—0.394
2	—0.280	—1.620	15	—0.411	+0.359	28	—0.656	+1.051	41	—0.226	—0.620
3	+0.406	—1.214	16	—0.430	—0.071	29	—0.161	+0.940	42	—0.562	—1.182
4	+0.515	—0.699	17	—0.255	—0.326	30	—0.031	+0.971	43	—0.238	—1.420
5	+0.957	+0.258	18	+0.463	+0.137	31	+0.448	+1.419	44	0.000	—1.420
6	+0.741	+0.999	19	—0.287	—0.150	32	+0.247	+1.666	45	—0.686	—2.106
7	+0.570	+1.569	20	—0.114	+0.264	33	—0.035	+1.631	46	—0.144	—2.250
8	+0.076	+1.645	21	—0.316	—0.580	34	—0.269	+1.632	47	—0.317	—2.567
9	+0.403	+2.048	22	+0.473	—0.107	35	—0.333	+1.029	48	—0.078	—2.645
10	—0.302	+1.764	23	+0.381	+0.274	36	—0.101	+0.929	49	+0.046	—2.569
11	—0.191	+1.555	24	+0.257	+0.531	37	—0.352	+0.577	50	+0.030	—2.569
12	—0.549	+1.006	25	+0.381	+0.912	38	—0.537	+0.030	51	+0.400	—2.169
13	—0.329	+0.677	26	+0.622	+1.534	39	—0.385	—0.345	52	+0.267	—1.902

No. 17) is represented by Diagram No. 12. To show the peculiar correspondence between the loan and the reserve periods, the latter is represented by the dotted line.

In the loan period, the upper extremes occur about the weeks ending March 4th and July 8th, and again August 13th–19th. The low points come during the weeks ending January 14th, April 29th and May 27th. A considerable recession occurs from July 8th to July 29th. The low point of the year falls about December 2d.

§ 45. The *quarterly period of the loans* (Table No. 18) is represented by Diagram No. 13 (A). The average weekly changes for the quarterly period were derived from the weekly changes of the annual period, which are simply divided into four parts. The indices for each of the thirteen weeks showing the quarterly movement of the fund are obtained as in the former periods by correcting the changes and subtracting the quarterly average.

TABLE No. 18.

Quarterly and Monthly Periods of the Loans.

Week.	Quarterly Period.	Week.	Quarterly Period.	Day of Month.	Monthly Period.
1	+0.103	8	+0.215	5	+0.126
2	−0.289	9	+0.165	12	+0.073
3	−0.472	10	+0.203	19	+0.022
4	−0.481	11	+0.083	26	−0.045
5	+0.030	12	−0.061	30	−0.074
6	+0.216	13	−0.049		
7	+0.292				

The period is only suggestive, for the truly representative period should be based on the deviations from the interpolated form* which should best fit the annual period.

As a suggestion period, it is significant. The first month shows a declining tendency, with the minimum about the end of the fourth week. During the next two weeks there is a rapid rise of about 0.8. This is in anticipation of the mid-quarterly settling date. The next weeks show a declining tendency with a slight advance for the first week of the third month. During the eleventh and twelfth weeks, there are rapid declines. The thirteenth week shows an advance in anticipation of the next quarter.

This is roughly true to the tendency. The indices are, however, more suggestive than quantitative. The 0.8 increase during the fifth, sixth and seventh weeks amounts at the present time to only about six and one-half millions of

* A form of the sine curve (Poisson) would, perhaps, be statistically convenient.

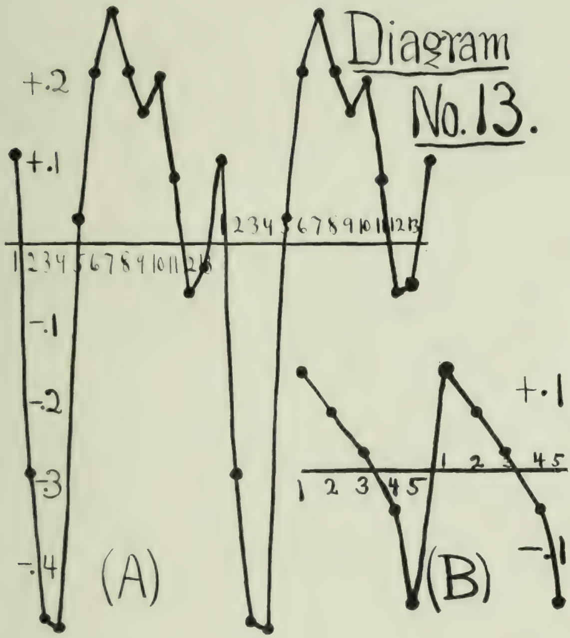


Diagram No. 13.

dollars in money. How this period may be made quantitatively representative will appear later (§ 68).

§ 46. The monthly period may be suggested roughly from five dates. The dates are the 5th, 12th, 19th, 26th and 30th. The position of the fund at each date (Table No. 18) is measured from an average in the manner previously explained. Like the quarterly period, the monthly period (Diagram No. 13 (B)) is suggestive rather than quantitatively representative. It consists, however, of the averages of a large number of weeks and is, it is safe to say, true to the tendency.

It is plain that the maximum occurs in the early part of the month. The average for the first date, which is the 5th of the month, is the largest, and a declining tendency ensues up to the last record, which is about the 30th of the month. There is an advance between the 30th and the 5th of 0.3, or at the present value of the growth axis of the loans, of about two and one-half millions of dollars.

The chart is interesting in that it displays the habit that business men have formed of borrowing money at the first of the month and paying off gradually up to the close. This is, in distinction from the annual period of the reserves, not dependent upon the facts of nature, but upon the customs of men acting with reference to an artificial division of time.

CHAPTER VI

CORRELATION

§ 47. Before commencing the subject of correlation, it is necessary to introduce two additional statistical tables. Table No. 19 contains the average weekly discount rate on call loans at the New York Stock Exchange from 1885 to 1900. These averages from 1887 to 1900 have been collected from the files of the Financial Review. The two earlier years, 1885 and 1886, have been collected from the editorials of the Commercial and Financial Chronicle.

This series of the Chronicle is probably the most reliable array of call rate statistics in existence. There exists another series of bank call discounts, which during the earlier years may be found printed on the weekly statements of the New York Clearing House. These statistics are, however, thoroughly unreliable, and the practice of publishing such a weekly rate has now for a good many years been abandoned by the Clearing House.

The call discount rate at the Stock Exchange is probably of all barometers the most sensitive to the immediate money market. The rate at the Stock Exchange is even more sensitive than the call rates at the banks. The reason for this lies in the fact that "bank loans are not called so soon as the loans on the Exchange and hence always command a slightly higher rate."*

The result inevitably forces the quickest changes upon the rate at the Stock Exchange and makes it the barometer of the money market at the time rather than an average rate governing an anticipated period of thirty to sixty days.

The range of the rate at the Stock Exchange is very wide. The rate has at times fallen as low as three-quarters of one per cent. for several days at a time. At the banks, on the other hand, the practice is, when money becomes superfluous,

* Wall Street Journal, January 30, 1901.

TABLE NO. 19.

Weekly Average Call Discount Rate at the Stock Exchange.

Week.	Date.		1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
			−4	−5	+1	0 −1	−2	−3	−4	−5 −6	0	−1	−2	−3 −4	+2	0	−1	−2
1	Jany.	7	*1¼	*3	5	5	5½	20	5	2½	5	1⅛	1¼	7	2	3½	3⅛	6
2		14	*1	2	5	4	3	6	4½	3	4½	1	1¼	5	1¾	2¾	2¾	5
3		21	*1½	*1¾	5	4	2½	5	4	2½	3½	1	1⅛	4½	1¾	2	2½	3¾
4		28	*1½	2	4¾	3	2¼	4	3	2	3	1	1⅛	4	1⅝	1¾	2½	3¼
5	Feby.	4	*1	1¾	4	3	2	3½	3	2	2	1	2	4	1⅝	1⅝	2⅜	2¾
6		11	*1	1¾	4	2½	2½	4	3	2	2½	1	1½	4½	1⅝	1⅜	2½	2¼
7		18	*1¾	*1⅝	4	2½	2	4	3	2	3½	1	1¾	4	1⅝	1⅜	2½	2¼
8		25	*1	*1⅞	3½	2¼	2	4	2½	2	4	1	1¼	3½	1⅝	2¾	2½	2¼
9	March	4	*1	2½	3½	2½	2¼	5	3	2	6	1	1½	3¾	1⅝	2¼	2¾	2¼
10		11	*1	2¼	4	3	2½	4½	3	2	15	1	2	3½	1⅝	2	2¾	2¾
11		18	*1	2	4	2½	2½	4½	2½	2	9	1⅛	2¼	3¼	1⅝	2⅛	4	5
12		25	*1	2⅛	4½	2½	2½	4	3	2	4	1⅛	2¼	4	1⅝	2⅛	5	4½
13	April	1	*1	3	5	2½	3½	4	3	2	7	1⅛	2½	3¼	1⅝	2⅜	6	3½
14		8	*1	3	6	2½	4½	4½	3	2	5	1⅛	2½	3½	1⅝	2⅞	7	3¾
15		15	*1	2⅜	6¾	2½	3½	4½	3	2	4½	1⅛	2½	3¼	1½	3½	4¾	3¼
16		22	*1	*2	6	2	3	4	3½	2	5	1⅛	2¼	3	1½	3	4¾	3
17		29	*1	*2¾	4½	2½	2¾	4	3½	2	5	1⅛	1¾	2¾	1⅜	2½	4	2¼
18	May	6	*1	2	5	2	2¼	4½	3½	2	6	1⅛	1½	2⅝	1½	3	4½	2
19		13	*1	2¼	5½	2	2½	5	4	1½	4	1	1⅜	2¾	1⅜	2¼	4	2⅛
20		20	*1	2	5	1¾	2	5	4½	1½	3	1⅛	1¼	3	1⅜	1¾	3½	2
21		27	*1	1¾	5	1½	2	5	4½	1½	2½	1⅛	1¼	2⅜	1⅜	1⅜	3	2
22	June	3	*1	2¾	5	1½	2½	4½	4½	1½	2½	1⅛	1¼	2	1¼	1⅜	2⅝	1⅞
23		10	*1	*2½	4	1⅜	2½	5	4	1¼	4½	1	1⅛	1¾	1¼	1¼	2⅜	1⅞
24		17	*1	2¼	5	1½	2½	4½	3½	1¼	7	1	1⅛	1¾	1⅛	1¼	2⅜	1⅞
25		24	*1	2	7	1½	2	4½	3	1¼	9	1	1⅛	2½	1⅛	1¼	2¼	1⅝
26	July	1	*1	2	10	1½	3½	5	2½	1½	15	1	1¼	1¾	1¼	1¼	3½	1⅜
27		8	*1¼	3	6	1½	3½	5	3	1¾	8	1	2	2⅜	1⅜	1¼	5	1¾
28		15	1	2	5	1½	3½	5	2½	2½	8	1	1⅜	1¾	1⅛	1¼	5	1⅝
29		22	1	2	4½	1½	3	4½	2	2	5	1	1¼	2¼	1⅛	1¼	4	1⅜
30		29	*1½	2	4½	1½	2¾	4½	2	1½	10	1	1¼	2½	1⅛	1¼	3⅞	1⅜
31	Aug.	5	1½	2¼	4½	1½	2¾	4	1½	1½	9	1	1⅛	1¾	1⅛	1¼	4	1⅜
32		12	1¾	2¾	4½	1½	3	5½	1¾	1½	5	1	1⅛	3¼	1⅛	1⅜	3½	1¼
33		19	1¾	6	5	1½	3½	10	2	1½	4	1	1	5	1⅜	1⅝	2⅞	1⅜
34		26	1½	7½	5½	1½	5	25	2¼	1¾	5	1	1	4½	1⅜	2	2⅞	1⅜
35	Sept.	2	1½	6¾	5½	1½	4½	6	2½	2	4½	1	1	6	1¼	2¼	3⅛	1¼
36		9	1½	6½	*5¾	2	3½	6	3½	3½	4	1	1	6½	1⅜	3½	5	1¼
37		16	*1¼	5¾	6	*2½	3½	10	4	4	4	1	1½	5½	2¼	3⅝	6	1½
38		23	*1¼	5¼	5	2	4½	7	3	4½	3	1	2	6	3¼	4¼	6½	1¾
39		30	1½	6	5	2	5½	4	6	3½	4	1	1¾	4¾	3	3¾	8	1 15/16
40	Oct.	7	1¾	*6¼	5	3	8	4	6	4½	3	1	2	4½	3	2¾	12	2⅜
41		14	*1¾	7	4½	2½	7	4½	5½	5	2½	1	2¼	6½	2¾	2½	7	3¼
42		21	1¾	6½	4	2	8	4½	4½	6	2	1	2¼	6½	2¼	2	5¼	3¼
43		28	2½	6½	3½	2	8	6	3½	6	2	1	2⅛	6½	2	1¾	6	4½
44	Nov.	4	*3	5	3½	2	8	6	3½	5½	2	1	2¼	25	2	1¾	10	4½
45		11	2⅝	7	*4¼	2½	5½	6	5½	6	2	1	2⅛	15	1¾	2	9	9
46		18	2½	5½	4½	2¼	5½	8	4½	5¾	1½	1	2	4	1¾	2¼	7	4
47		25	2¾	5½	5	2½	6	8	4	5	1½	1	1¾	3½	1¾	2¼	6	3¾
48	Dec.	2	*2⅝	8	*5¼	2½	6	6	3½	4½	1½	1⅛	2	2½	1⅞	2¼	6	3½
49		9	*2¼	7	4¾	2¾	6	6	3	4½	1¼	1⅛	2¼	2¼	1¾	2¼	7	4½
50		16	2	7	5	3	6	6	3	4¾	1¼	1½	2	1¾	3½	2⅜	7	5½
51		23	*2½	25	4½	4	6	4	2¾	7	1	1½	4	1¾	3¾	2¼	25	5¼
52		30	*2½	6	5	*5	7	4	3	10	1⅛	1⅝	10	2	3¾	2¾	5½	5¼
				5½						5½				2				

to make "a uniform rate of 3% for all periods. Banks do not like to go below 3%, because they reckon the cost of money to them, including reserve requirements, at about 2 and ⅓%."* The upper limit of the call rate at the Stock Exchange is around 200%. A rate of 186% occurred during the panic of December 1899, and has been registered at several other such disturbances in credit.

In the earlier years the average was estimated from daily fluctuations for some dates. These dates are marked in the table by asterisks.

§ 48. The second additional table (No. 20) contains the weekly ratios of reserves to deposits. These ratios are published in the Clearing House weekly statements. The table was compiled from the files of the Financial Review for the years 1885 to 1900.

§ 49. Of the several classes of problems outlined in Part I (§ 7), we have already considered "changes in one column" of the media of exchange expression "coincident with the passage of time" under the head of Growth Axes. "Changes in one column coincident with recurrent periods of time," we have treated to some extent under the head of Periodicity. We now come to the third division, "*correlation* between items in different columns." This correlation from the standpoint of time, may be either *immediate* or *anticipatory*.

The nature of this class of problems is best seen from a concrete example. Let us take the movements of three banking items on the Chart. These items are the average weekly discount rate on call loans as reported by the Commercial and Financial Chronicle since 1885 (Table No. 19), the ratios of reserves to deposits (Table No. 20), and finally the array of reserve deviations (Table No. 8). The Chart is properly suggestive. The heavy black line represents the reserve deviations, the dotted line the call loan rate and the smooth black line the ratio of reserves to deposits, with the straight line beneath it the 25% limit required by law.

* Wall Street Journal, January 30, 1901.

TABLE NO. 20.—*Ratios of Reserves to Deposits (1885–1900).*

Week.	Date.	1885	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
		−4	−5	+1	0 −1	−2	−3	−4	−5 −6	0	−1	−2	−3 −4	−5	+1	0	−1
1	Jan. 7	36.7	31.4	29.0	27.9	26.8	25.4	27.2	28.7	27.0	41.2	31.5	29.0	31.3	28.3	27.8	26.5
2	14	38.6	32.4	30.0	29.2	28.5	26.5	28.4	29.0	28.3	42.5	32.5	30.5	33.0	28.6	28.4	27.2
3	21	39.5	33.5	30.0	30.6	29.3	26.9	29.8	30.0	29.7	43.9	33.1	31.6	34.4	29.3	29.1	28.1
4	28	39.8	34.0	30.8	30.4	29.7	28.5	30.9	31.6	29.7	44.9	33.2	32.7	35.0	29.9	29.5	28.7
5	Feb. 4	40.3	34.3	30.6	30.9	29.4	28.4	30.8	32.0	28.8	45.2	31.7	33.1	34.6	29.7	29.2	28.8
6	11	40.5	34.0	33.3	30.2	28.3	27.3	29.9	31.5	28.5	41.3	30.2	33.2	33.3	29.4	28.9	28.4
7	18	39.5	33.7	29.0	29.6	28.9	26.7	29.4	31.5	27.8	39.1	31.3	32.5	34.1	28.5	28.8	27.9
8	25	39.1	33.3	32.8	29.0	28.6	25.9	28.7	30.8	27.0	39.0	30.6	30.9	32.8	28.1	28.3	27.3
9	Mar. 4	39.0	31.6	27.4	28.4	27.8	25.6	28.3	30.0	26.5	39.3	30.3	30.0	35.0	27.9	27.7	26.6
10	11	38.4	30.9	27.1	28.0	26.9	25.1	27.6	29.0	26.0	39.2	29.3	29.7	34.6	28.3	27.5	25.6
11	18	38.2	30.2	26.6	27.7	26.8	25.3	27.4	28.0	26.4	39.4	28.4	29.4	34.1	29.1	27.1	25.3
12	25	38.5	29.4	26.7	27.5	26.5	25.8	27.2	28.0	27.1	39.5	27.8	28.9	33.4	29.9	27.0	25.7
13	April 1	38.6	28.4	26.2	27.4	26.5	26.1	27.0	28.4	27.4	40.3	27.7	28.7	33.5	30.3	26.7	26.2
14	8	38.4	28.2	26.1	27.3	25.3	25.4	26.5	28.7	27.1	39.6	27.8	28.5	33.4	30.2	26.7	25.9
15	15	38.7	29.2	26.2	27.9	26.4	25.1	26.3	28.1	27.5	39.3	28.0	28.7	33.1	30.6	27.2	26.3
16	22	38.1	29.9	26.8	28.8	27.7	25.3	26.5	28.0	28.3	39.4	28.9	28.8	33.3	31.6	27.7	26.7
17	29	39.9	28.6	26.9	29.3	28.0	25.8	26.7	28.7	27.8	39.6	29.9	29.2	33.6	31.8	27.9	27.0
18	May 6	40.4	28.2	26.2	29.7	27.2	25.8	26.8	28.7	28.0	39.3	30.2	29.6	33.5	31.7	27.1	26.8
19	13	40.3	28.0	34.5	30.7	27.0	25.4	26.2	27.8	29.1	38.9	30.8	29.1	34.5	32.1	28.0	26.7
20	20	40.7	28.4	26.2	32.1	28.2	25.5	26.2	28.0	30.6	38.7	31.8	28.8	32.8	32.6	28.8	26.9
21	27	41.5	28.8	26.6	32.2	28.4	25.9	26.3	28.7	30.8	38.5	32.1	29.4	33.0	32.7	29.9	27.1
22	June 3	41.8	28.8	26.2	31.6	28.3	26.2	26.9	30.0	29.8	38.6	32.3	29.4	33.1	32.4	29.8	27.2
23	10	41.5	28.3	26.2	31.9	27.3	26.0	26.7	29.4	28.4	38.4	32.0	28.7	32.4	32.4	29.3	27.0
24	17	42.1	29.0	26.2	32.0	27.4	26.7	27.7	29.3	27.1	38.4	31.7	29.1	33.1	33.1	28.3	26.9
25	24	41.8	29.2	25.9	31.8	27.1	26.5	29.1	29.2	26.4	38.3	31.4	29.0	33.2	33.4	27.8	26.7
26	July 1	41.8	28.9	26.0	31.6	26.5	26.6	29.6	29.0	25.3	38.0	31.0	29.1	33.2	33.3	26.6	26.9
27	8	41.2	27.9	25.6	30.9	26.1	25.9	28.8	28.4	23.7	37.4	30.6	29.1	31.8	31.9	25.5	26.7
28	15	41.7	28.5	27.2	31.6	26.5	26.5	28.6	27.9	23.9	37.5	30.9	29.4	31.8	31.5	26.2	27.2
29	22	41.6	28.6	27.3	31.4	26.6	26.4	29.5	29.0	24.4	37.6	31.8	29.3	32.5	30.8	26.4	27.7
30	29	41.7	28.8	27.2	31.6	26.6	26.5	29.8	29.4	23.9	37.3	32.4	28.7	32.6	30.6	26.2	28.1
31	Aug. 5	41.9	28.4	26.9	31.5	26.9	27.2	29.8	29.6	21.2	36.9	32.1	28.6	32.3	30.3	25.9	28.2
32	12	40.9	27.3	26.4	30.8	26.6	25.3	29.6	28.6	20.6	36.5	31.9	27.9	31.5	29.3	26.7	28.1
33	19	40.3	27.0	26.2	30.2	25.1	24.8	29.3	28.0	21.7	36.6	32.1	27.0	31.2	28.8	26.8	27.3
34	26	39.9	26.9	26.4	30.1	25.5	24.4	28.5	27.4	23.1	36.4	31.5	27.0	31.3	27.9	26.4	27.6
35	Sept. 2	39.5	26.3	26.5	29.1	26.1	24.9	28.2	26.9	24.6	36.2	31.8	26.9	31.2	27.0	26.1	27.9
36	9	38.3	27.0	31.9	27.9	26.9	24.6	27.3	26.5	25.8	35.0	31.0	26.8	31.9	26.0	25.3	27.8
37	16	37.8	27.2	26.1	27.5	26.2	24.1	27.2	26.0	27.8	35.2	29.7	26.9	29.2	25.6	25.0	27.3
38	23	37.2	27.2	26.7	27.9	25.5	25.5	26.9	26.0	29.6	35.2	29.0	27.3	28.1	26.1	25.4	26.8
39	30	39.3	27.6	27.6	28.6	25.2	28.5	26.0	26.1	31.2	35.4	29.1	28.2	27.6	27.2	25.2	26.4
40	Oct. 7	35.8	26.7	27.3	27.8	24.6	27.8	25.8	26.0	32.1	35.1	28.0	28.6	27.5	27.6	25.0	25.2
41	14	35.4	27.6	27.3	27.8	24.8	25.8	26.6	25.4	33.4	35.3	27.6	28.4	27.2	27.7	25.3	25.5
42	21	34.4	26.3	27.6	29.0	25.2	24.0	27.2	25.1	35.1	35.5	27.9	27.6	27.3	28.1	25.2	25.3
43	28	33.1	26.6	28.3	28.8	25.3	25.0	28.0	25.5	36.3	35.7	28.2	28.3	28.7	28.4	25.4	25.7
44	Nov. 4	32.4	26.7	27.7	28.3	25.3	25.2	28.0	25.9	36.5	35.6	28.3	29.9	28.9	27.5	24.9	25.6
45	11	31.7	26.6	27.7	27.8	24.6	24.4	26.2	25.6	37.7	35.4	28.7	28.4	28.4	26.9	24.6	25.5
46	18	32.0	27.2	27.1	27.8	25.1	24.8	27.3	25.6	39.1	35.6	28.8	30.2	28.4	27.1	24.9	25.9
47	25	32.2	27.8	28.4	28.0	25.4	24.1	28.2	26.0	39.9	36.2	28.7	32.6	28.6	27.4	25.9	26.4
48	Dec. 2	32.0	27.5	26.7	27.5	25.5	25.1	28.0	26.5	40.6	34.0	28.5	33.2	28.4	27.1	26.1	26.2
49	9	32.0	26.7	26.8	26.8	25.2	24.4	28.2	26.4	40.5	30.8	28.9	31.4	28.3	27.1	25.9	25.6
50	16	32.6	26.1	27.3	27.4	25.7	25.2	28.5	26.2	40.4	31.9	28.5	31.1	27.8	27.1	25.9	25.7
51	23	32.1	26.1	27.6	26.8	25.8	26.2	29.1	26.2	40.6	31.1	28.3	31.3	27.3	27.4	26.2	26.1
52	30	30.3	27.1 28.4	27.4	26.6	25.5	27.0	29.3	26.4 26.5	41.0	31.4	28.1	31.5	26.9 25.0	29.8	26.5	26.3

Glancing at the continuous movement of these lines in relation to each other, three points may be remarked:—

(1) That the reserve deviations and call discounts vary inversely;

(2) That the ratio of reserves to deposits and call discounts vary inversely;

(3) That the inverse variation increases in amplitude as the ratio of reserves to deposits approaches the legal ratio of 25%.

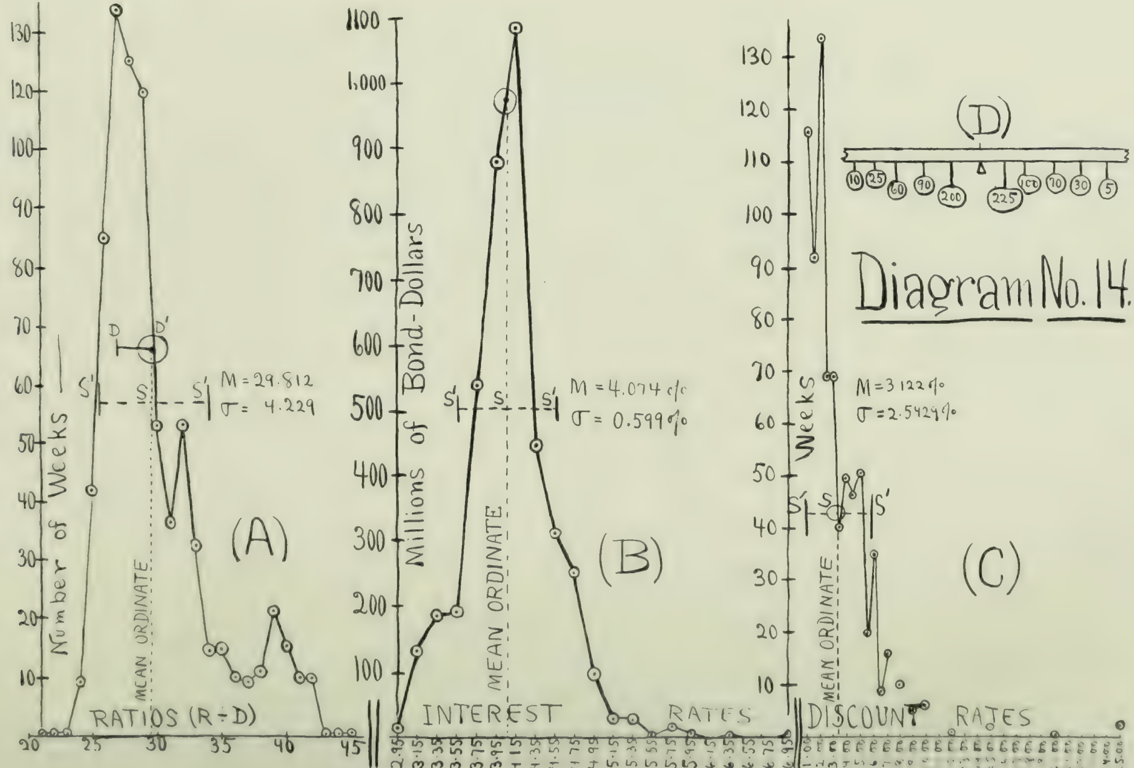
These co-variations of two or more items are properly correlations. The Chart suggests that correlation exists. The suggestion is simply one of fact and not quantitative. Hence the following questions at once arise. To what degree are these variations correlated? Is one correlation greater than another? If we knew next week's value for one item, what value could we predict for the other and within what limits? All such questions the diagrams do not answer.

A method is needed to work out the quantitative measure of correlation. Such a method has been invented for use in biology by Prof. Karl Pearson of London. It has not been used to any extent in economics.* Consequently a brief exposition of the method may not be out of place.

§ 50. In constructing the *coefficient of correlation*, two quantities must be calculated for each set of statistics. These quantities are the *mean* and the *standard deviation*. The last two quantities are very important constants for the frequency curve of any array of statistics. Hence it is important to know at the outset exactly what is meant by these several terms.

Concrete examples of frequency curves are the distribution of the number of weeks from 1885 to 1900 according to the ratio of reserves to deposits, the distribution of bond-dollars according to the actual rate of interest realized at the market price, or the distribution of the number of weeks from 1885 to 1900 according to the call discount rate.

* A few exceptions to this statement may be noted, as Yule, on *Pauperism* (Roy. Stat. Soc., 1899, Vol. 62, pp. 249–286).



Number of Weeks
RATIOS (R ÷ D)
MEAN ORDINATE
M = 29.812
σ = 4.229
(A)
Millions of Bond-Dollars
INTEREST RATES
M = 4.074 %
σ = 0.599 %
(B)
Weeks
DISCOUNT RATES
M = 3.122 %
σ = 2.5429 %
(C)
(D)
Diagram No. 14.

TABLE NO. 21.

Frequency of Bond-Dollars to Actual Yield, in Millions.*

Actual Rate of Interest.	Bond Dollar Frequency.	Actual Rate of Interest.	Bond Dollar Frequency.	Actual Rate of Interest.	Bond Dollar Frequency.	Actual Rate of Interest.	Bond Dollar Frequency,
2.85–3.04	13	4.05–4.24	1088	5.25–5.44	26	6.45–6.64	0
3.05–3.24	131	4.25–4.44	444	5.45–5.64	2	6.65–6.84	0
3.25–3.44	183	4.45–4.64	311	5.65–5.84	14	6.85–7.04	4
3.45–3.64	190	4.65–4.84	251	5.85–6.04	2	total	4204
3.65–3.84	540	4.85–5.04	96	6.05–6.24	0	mean rate	4.0736
3.85–4.04	881	5.05–5.24	27	6.25–6.44	1	S. D.	0.5990

Thus in Diagram No. 14 (A) we lay off upon the horizontal axis at equal intervals the numbers or ratios 20%, 21%, 22%, ... 45%, which are the ratios of reserves to deposits. At each ratio of reserves to deposits, perpendiculars are erected proportionate to the number of weeks with that particular ratio during the years 1885 to 1900. The outline joining the tops of these perpendiculars is the so-called *frequency polygon.* In the bond-dollar frequency array (Diagram No. 14 B), the base line represents the actual rate of interest and the ordinates bond-dollars. In Diagram No. 14 (C), the abscissas signify discount rates and the ordinates number of weeks. Every such frequency polygon has a number of constants, which are of very great importance in the characterisation of the statistics. Among these constants, the most important are the mean, the mode and the standard deviation.

The *mean* is what is commonly called in economics the weighted average. Its formula is $\frac{\Sigma x.y}{n}$ where x is the abscissa quantity, y the ordinate quantity and n the number of records. In physics, the mean is known as the abscissa of the centre of gravity. Upon the Chart, the circle marks the height of the ordinate to the mean, and where the line cuts the horizontal axis lies the mean of the ratios of reserve to

* The above table is compiled for bonds listed on the New York Stock Exchange about November 1, 1901. Data for the actual yield were gathered from the Wall Street Journal. The array represents about $4,200,000,000 of capital invested in railroads, which were paying interest regularly at the time.

deposits (29.8115). The *mode** represents the highest peak of the polygon, i. e. the class of greatest frequency. In this case it is about 27.

Some method† of appreciating how the "deviations are distributed along the range" is our next consideration. The diagram suggests to the eye a certain representation, but it is again only suggestive, not a measurement. The notion of a "numerical value of the variation" may be appreciated from the following example in physics.

If a bar hung with weights (Diagram No. 14, D) be set "rotating on the given rough pivot at a certain speed, friction will bring it to rest in a given time."

"Now the greater the concentration of weights about the pivot, the sooner the bar comes to rest; the further out from the pivot the weights are, the longer it takes to come to rest. In other words, *the time the bar takes to come to rest is a measure of the concentration or scattering of the weights along the range.*"

"Now physicists tell us that this time is proportional to the square of a certain quantity termed the spin or swing radius, and which I will denote by the Greek letter (σ)."

"σ is then shown to be the mean of the squares of all the individual deviations, and in our quantitative study of evolution σ is termed the standard deviation."

"Other measures of variability have been devised, some of which are occasionally useful, but for theoretical and practical reasons the standard deviation may be considered the best. It is not hard to find, and it occurs and recurs in all sorts of investigations."‡

The *Standard Deviation* (σ) is neither more nor less than the square root of the weighted average of the squares of the deviations of the different classes from the mean. The formula is

$$\sigma = \sqrt{\frac{\Sigma v^2}{n}}.$$

* The exact value must be determined by interpolation.

† Pearson's *Grammar of Science*, p. 386, etc.

‡ See *Grammar of Science*, p. 387.

Pearson offers the following rule for calculating the standard deviation. "Multiply the frequency with which each individual type occurs by the square of its deviation from the mean; add all the products together and divide by the total number of individuals. This is the square of the standard deviation."

The standard deviation in the first frequency polygon is 4.2285 ± 0.0722 and is represented by the distance SS′, measured from the mean upon the chart. The distance SS′ represents an average concentration of deviation. In the second frequency polygon, σ is 2.5429 ± 0.0434, and in the third, σ is 0.5990 ± 0.00004.

The distance which separates the mean from the mode, DD′ on the diagram, signifies that the curve is skew or asymmetrical. The measure of skewness is the distance (DD′/SS′), or, in words, the difference between the mean and the mode divided by the standard deviation.

Another factor descriptive of the statistics is the range. Is the range limited or unlimited? Pearson says,*—"to deal effectively with statistics we require generalised probability curves which include factors of skewness and range." . . .

"Accordingly, we require the following types of frequency curves":—

"Type I.—Limited range in both directions, and skewness.

"Type II.—Limited range and symmetry.

"Type III.—Limited range in one direction only and skewness.

"Type IV.—Unlimited range in both directions and skewness."

"Type V.—Unlimited range in both directions and symmetry."

Pearson shows that† "Types I., II., III., and V. are all represented by the curve

$$y = y_0 (1 + x/a_1)^{va_1} (1 - x/a_2)^{va_2}$$

* Phil. Trans., Vol. 186, Part I, p. 360.

† Phil. Trans., Vol. 186, Part I, p. 367.

and Type IV. by the curve

$$y = y_0 \frac{1}{(1 + x^2/a^2)^m} e^{-v \tan^{-1} x/a}."$$

In so hasty a sketch, it is sufficient to remark that Pearson has developed simple criteria by which the frequency polygon, if of homogenous material, may be easily fitted with a smooth curve of appropriate type.* The difference between the shape of the first two frequency polygons and the shape of the third (Diagram No. 14) shows how *inadequately* the means alone describe the character of the statistics. In C, the highest frequency occurs almost at the start.

§ 51. The philosophy underlying Pearson's work is quite at variance with the commonly accepted theory, that normal distribution applies to the errors of observation, a scientific dogma which has been used for many years in all kinds of physical and astronomical measurements.

Indeed the notion that one certain type, namely, the normal curve of errors, represents a natural frequency of distribution in the world of phenomena, has in it to the average mind no rationale. The notion has had of late a decided set-back.†

* Frequency polygons are of many forms in practical statistics. Thus the following eccentric form sometimes occurs, "an asymmetrical curve with limited range, mediocrity being in a minimum. The disappearance of mediocrity is not a very uncommon feature of statistics; the 'prevalence of extremes' may appear not only in meteorological phenomena but in competitive examinations, where the mediocre have occasionally sufficient wisdom to refrain from entering." (Pearson, cited above, p. 365.)

† Says Pearson in a recent issue of the Philosophical Magazine (1900), after criticising the above notion in detail, "It is desirable to illustrate such results a second time. Prof. Merriman in his treatise on Least Squares starts in the right manner, not with theory, but with actual experience, and then from his data deduces three axioms. From these axioms he obtains by analysis the normal curve as the theoretical result. But if his axioms be true, his *data* can only differ from the normal law of frequency by a system of deviations such as would reasonably arise if a random selection were made from material actually obeying the normal law.

"Now Professor Merriman puts in the place of honour 1000 shots fired at a line on a target in practice for the United States Government, the deviations being grouped according to the belts struck; the belts were drawn on the target of equal breadth and parallel to the line. . . . . .

"Hence we deduce" . . . . "P = 0.000144."

"In other words, if shots were distributed on a target according to the normal

Pearson further shows that even the throws of dice do not follow the laws of chance—i. e. normal distribution, and that the returns from the roulette tables of Monte Carlo* are decidedly at variance with the scientific canons of probability.

§ 52. With this rough sketch of the notions underlying the frequency curve,† we are ready to pass to the subject of correlation. This section might seem, on first view, the proper place to introduce an abstract exposition of the method, but owing to the ever necessary caution which should attend a writer on an economic subject, requiring a certain amount of mathematics, not to subordinate the economics to the mathematics, it has seemed best to treat the method only as it became necessary for clearness in the development of the economic subject.

The three ensuing chapters will consider three correlations between economic quantities.

law, then such a distribution as cited by Mr. Merriman could only be expected to occur, on an average, some" . . . "14 or 15 times in 100,000 trials." "Now surely it is very unfortunate to cite such an illustration as the foundation of those axioms from which the normal curve must flow."

See Pearson, *On the Criterion that a given System of Deviations from the Probable in the Case of a Correlated System of Variables is such that it can be reasonably supposed to have arisen from Random Sampling.* Philosophical Magazine, July 1900, p. 157, and June 1901, p. 670.

* Pearson, *The Chances of Death and Other Essays*, p. 42.

† It has seemed best to omit discussion of the equations of the frequency curves of interest rates and bond-dollars in this volume, and to confine the scope to the less technical subject of correlation.

CHAPTER VII

CORRELATION BETWEEN THE CALL DISCOUNT RATE AND THE RATIO OF RESERVES TO DEPOSITS

§ 53. A glance at the Chart discloses to the eye the already noted fact, that inverse correlation exists between the ratio of reserves to deposits and the call discount rate.

What *a priori* reason is there in banking theory for this correlation? Banking theory is very full upon this point. The solvency of a bank must forever lie in the bank's ability to meet its just claims. To meet just claims at any time, a continuous reserve must be kept on hand. If, through coincidences in business, series of drains occur in succession, reducing the reserve, by so much is the solvency of the bank threatened.

The weapon of the banks in defense is the call discount rate. A bank loans less money, and what it loans is distributed to the applicants in loans at higher rates. Consequently we should expect just such a correlation. The framers of the National Banking System recognized this fact when they required a minimum reserve of 25%, the straight line upon the Chart. So far we simply verify an ancient banking theory.

We may now go further and ask what is the amount of correlation existing between these two factors. All the *a priori* reasoning in the world could never answer this inquiry. The answer must be derived from the facts. We have the facts for fifteen years, representing vast amounts of capital at one of the greatest financial centers of the world.

§ 54. In the following table (No. 22) are the calculated values of the *means* and *standard deviations* for ratios of reserves to deposits, call discount rates and actual rates of interest on bond-dollars.

TABLE NO. 22.

	Ratios of reserves to deposits.	Call discount rates.	Actual rate of interest on bond-dollars.
Mean (M)	29.8115 ± 0.1021	3.1220 ± 0.0614	4.0736 ± 0.00006
Standard deviation (σ)	4.2285 ± 0.0722	2.5429 ± 0.0434	0.5990 ± 0.00004

The mean ratio of reserves to deposits for these years has been very nearly 30%, and the average call discount rate about 3 and 1/8%. The measure of variability (σ) shows how extremely sensitive the call rate is. For comparison, the mean and standard deviation* of the actual rate of interest on bond-dollars are attached.

The difference between call loans and bond loans is in duration. The average duration for the former is approximately nine days; for the latter, the average† is in the neighborhood of forty-four years.‡ The actual rate of interest on call loans is slightly higher than the discount rate owing to the method of calculation by the banks,—about 3.222%. The mean call rate is thus .85% less than the mean actual rate on bond-dollars. The difference in variability is still more striking. Measured by the coefficients of variation $\left(\frac{\sigma}{m}\right)$ (§ 63), the call rate is over four times as variable as the bond rate.

* The probable errors of the mean and standard deviation are calculated by Pearson's formulæ,—P. E. $m = \pm 0.6745 \frac{\sigma}{\sqrt{n}}$, P. E. $\sigma = \pm 0.6745 \frac{\sigma}{\sqrt{2n}}$. In practice, it is easier to calculate these quantities after the correlation table has been prepared.

† It should be remembered that the ordinate quantity in the call discount polygon is number of weeks, and in the bond-dollar array millions of bond-dollars.

‡ The market value of the bond-dollar is simply the discounted value of a series of "futures" in the media of exchange. This series consists of the interest payments, semi-annually or annually to maturity, and the bond-dollar at maturity. The bond-dollar is the dollar promised at the end of a stated number of years, i. e. the duration. The actual rate of interest is the yield on the market price at the time, if the bond is held to maturity and all payments of interest as well as of the bond-dollar are realized. For the 4,204,000,000 bond-dollars selected, the distribution with respect to duration is as follows :—

1901–10	1911–20	1921–30	1931–40	1941–50	1951–60	1961–70	1971–80	1981–90	1991–2000
\$367M	\$446M	\$827M	\$669M	\$741M	\$219M	\$1 M	\$20 M	\$199M	\$715M

§ 55. Table No. 23 is the correlation table of call discounts and ratios of reserves to deposits. Along the upper edge, the call discount rates are numbered. Down the side, the ratios of reserves to deposits are entered. The number 30 in the square which reads upwards to 2% and towards the side to 28% means, of course, the number of weeks which combine a 2% rate with a 28% ratio.

TABLE No. 23.

Ratios.	Call Discount Rates.																			
	1	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	8.0	9.0	10.0	12.0	15.0	20.0	25.0
21	...	..	..	..	..	..	..	..	1	..	..	..	..	..	1	....	....	....	....	....
22	...	..	..	..	..	..	1	..	..	..	..	..	..	..	..	....	....	....	....	....
23	...	..	..	..	..	..	..	..	1	..	..	..	..	..	..	....	....	....	....	....
24	...	..	..	..	..	..	..	1	..	..	2	..	..	3	..	2	....	....	....	1
25	...	..	..	..	..	1	2	6	4	4	11	1	2	6	1	2	1	1	....	....
26	...	..	..	..	2	6	13	12	16	6	11	4	7	..	2	2	....	1	1	2
27	...	1	10	9	14	12	15	17	19	9	9	3	4	1	..	....	....	1	....	....
28	...	5	30	23	20	11	7	3	7	1	2	2	3	..	..	1	....	....	....	....
29	3	9	48	17	16	3	6	3	1	..	..	..	2	..	..	....	....	....	....	1
30	1	12	12	10	8	4	4	..	2	..	..	..	..	..	..	....	....	....	....	....
31	8	10	6	2	4	2	2	1	1	..	..	..	..	..	..	....	....	....	....	....
32	15	14	10	8	5		..	1	..	..	..	..	..	..	..	....	....	....	....	....
33	15	8	4	1	..	1	2	1	..	..	..	..	..	..	..	....	....	....	....	....
34	2	11	1	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
35	8	5	1	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
36	7	2	1	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
37	8		1	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
38	9	1	1	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
39	19	2	..	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
40	7	8	..	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
41	7	3	..	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
42	8	2	..	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
43	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
44	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....
45	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	....	....	....	....	....

§ 56. We are now prepared to form a measure of the amount of correlation. It does not matter in studying correlation whether the frequency polygons are normal or skew. Says Pearson: "In dealing with correlation and regression in such cases as this,* we must throw entirely on one side any notion of normal surface and curves of error and go simply

* Data for the *Problem of Evolution in Man*, Roy. Soc. Proc., 1899, Vol. 65, p. 294.

to the kernel of the affair. . . . What we want is the law connecting the mean age at death of one relative when another relative has died at a given age. When the given age of the latter and the mean age of the former are plotted to form a curve, this curve is the regression curve whatever be the form of the frequency surface."

If we make a table (No. 24), from the correlation table (No. 23), of the weighted averages of call discount rates for given ratios of reserves to deposits, we have a table of regression averages.

TABLE No. 24.

Regression Averages of Call Discount Rates on Ratios of Reserves to Deposits.

Ratios.	Frequency of Weeks.	Mean Discount Rate.	Ratios.	Frequency of Weeks.	Mean Discount Rate.	Ratios.	Frequency of Weeks.	Mean Discount Rate.
21	2	7.	29	109	2.71	38	11	1.23
22	1	4.	30	53	2.49	39	21	1.05
23	1	5.	31	36	2.13	40	15	1.27
24	9	9.50	32	53	1.80	41	10	1.15
25	42	6.46	33	32	1.67	42	10	1.10
26	85	6.05	34	14	1.46	43	1	1.00
27	124	4.30	35	14	1.25	44	1	1.00
28	115	3.14	36	10	1.20	45	2	1.00
			37	9	1.11			

Diagram No. 15 represents these regression averages by heavy dots. The broken line (k k′ k″) joining these dots is the regression polygon of call discounts on the ratio of reserves to deposits.

Two points may be remarked:—

(1) That call discounts increase as the ratio of reserves to deposits decreases;

(2) That the increase is not at a uniform rate, but that call discounts rise much more rapidly as the ratio approaches 25%.

§ 57. Now there are two kinds of regression curves, linear and skew. This series of regression averages belongs distinctly to the skew type. There are two general methods of fitting a mathematical curve to a skew regression. One is by the help of the method of least squares. The other is by the method of moments. The first is arithmetically much

more laborious than the second. Unfortunately, at this writing, Professor Pearson has not published the latter method.

The straight line (D E), crossing the diagram (No. 15) diagonally, is the coefficient of regression line on a linear basis. It is apparent that the fit is not very good. How the position of this line is determined will be discussed shortly.

To produce a better fit on the linear basis, I have broken the correlation table (No. 23) into two parts at the mean of the ratio of reserves to deposits (29.8115). I have fitted each portion of the table separately with a straight line.

This method Pearson finds very satisfactory in a recent memoir published in the Proceedings* of the Royal Society on the "Correlation between Duration of Life and the Number of Offspring." The left portion of Diagram No. 15, i. e. the number of records in the correlation table (No. 23) below the mean of the ratio (29.8), which involves two-thirds of the total number of weeks, is extremely well fitted by a straight line.

§ 58. To determine the slopes of these lines, it is first necessary to calculate the coefficient of correlation. The *coefficient of correlation* has for its mathematical formula the expression,

$$r = \frac{\Sigma Z(X - M_x)(Y - M_y)}{N\sigma_x \sigma_y}.$$

$Z =$ frequency of deviations $(X - Mx)$ and $(Y - My)$ from the means Mx and My in the total number of observations N. $\sigma_x =$ standard deviation of the X series and $\sigma_y =$ standard deviation of the Y series. In words, r is simply the quotient of the weighted average of the moments about the centre of gravity of the system (i. e. the point of intersection of the two means), divided by the product of the two standard deviations.

In perfect correlation $r = 1$. If no correlation exists, $r = 0$.

To illustrate this formula in the calculation of the coefficient of correlation for the ratio of reserves to deposits and call discounts for 780 weeks, the numerical values for the letters in the formula are as follows. The value of Σ, the

* Vol. 67, p. 164.

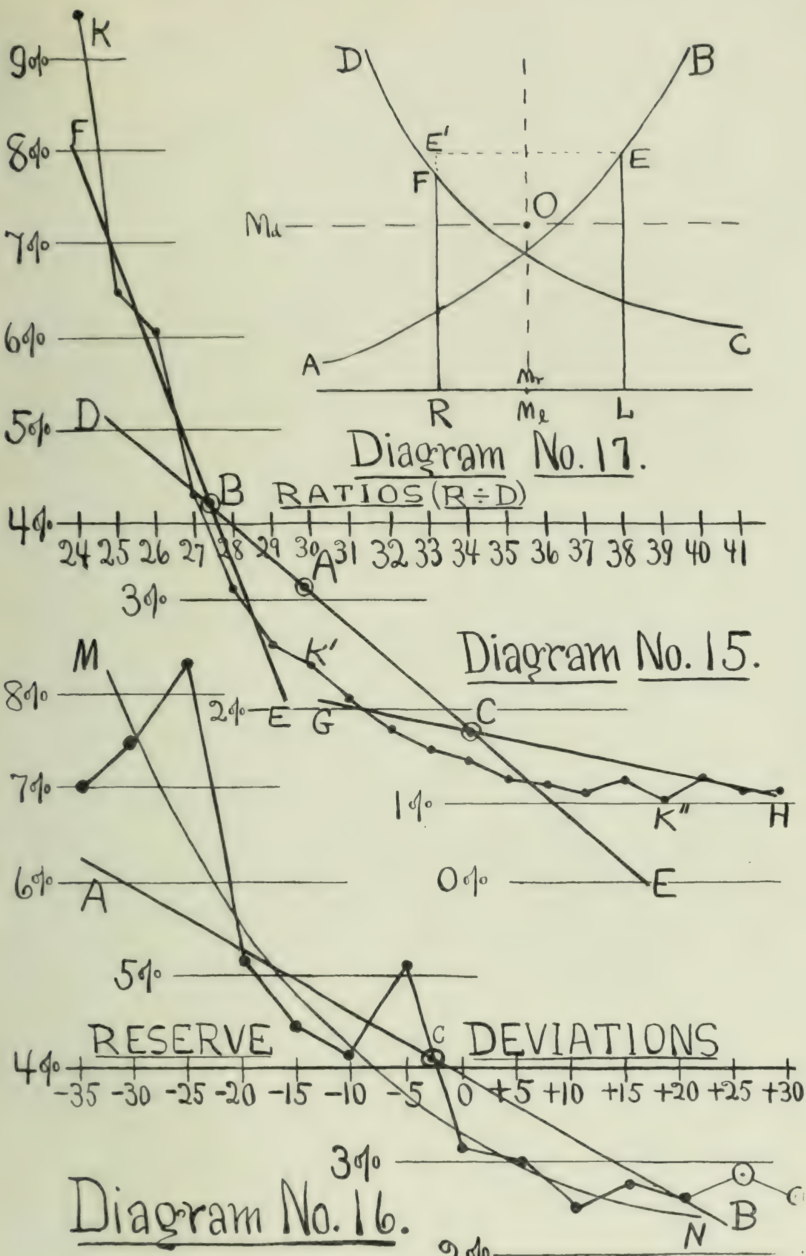


Diagram No. 17.

Diagram No. 15.

Diagram No. 16.

sum of the weighted moments, is 4,387.5. The number of weeks (N) is 780. The standard deviation of the discount rate (σ_d) is 2.5429 with the standard deviation of the ratio (σ_r) 4.2228. Inserting these values in the formula

$$r = \frac{4,387.5000}{(780)\,(2.5429)\,(4.2285)} = 0.5231 \pm 0.0142.$$

Thus r (0.5231) with a probable error* of 0.0142 is the quantitative index which measures the degree† of correlation existing between the ratio of reserves to deposits and the call discount rate, on the basis of 780 weeks of experience.

§ 59. In Diagram No. 15, the intersection of the means for the whole array of 780 weeks is marked A. The records were split at the mean of the ratio of reserves to deposits (29.8), and the number of weeks below the mean was found to be 488, and above the mean 292. The mean of the greater number of weeks is marked B upon the chart, and the mean of the minor number of weeks is marked C.

Now "the best fitting straight line‡ drawn through these means when plotted is the line of regression" of call discounts on the ratio of reserves to deposits, and "its slope" is $r\,\sigma_1/\sigma_2$.

The following table contains the means,§ standard devia-

* The probable error is calculated by the formula P. E. $= 0.67449\,(1-r^2) \div \sqrt{n}$. Cf. Yule, "On the Significance of *Bravais*' Formulae for Skew Correlation," 'Roy. Soc. Proc.,' Vol. 60, pp. 477–489.

† When the coefficient of correlation is six times the probable error, the correlation is considered very satisfactory. In this case, the coefficient is some 36 times the probable error.

‡ *Grammar of Science*, p. 400.

§ For convenience the formulas of m, σ, r and R are given below with their probable errors.

	Probable Errors.
Mean $(m) = \frac{\Sigma xy}{N}$.	$0.67449\,\sigma/\sqrt{n}$.
Standard Deviation $(\sigma) = \sqrt{\frac{\Sigma v^2}{n}}$.	$0.67449\,\sigma/\sqrt{2n}$.
Coefficient of Correlation $(r) = \frac{\Sigma(x-m_x)\,(y-m_y)z}{n\sigma_x\sigma_y}$.	$0.67449\,(1-r^2)/\sqrt{n}$.
Coefficient of Regression $(R_{12}) = r\sigma_1/\sigma_2$.	$0.67449\,\sigma_1/\sigma_2\sqrt{\frac{1-r^2}{n}}$.

Cf. *Pearson and Filon*, Phil. Trans. A, Vol. 191, 1898, pp. 229–311.

tions, coefficients of correlation and coefficients of regression for the total number of weeks, i. e. N = 780, and for the two divisions in which N = 488 and N = 292 respectively.

TABLE NO. 25.

		N=780 weeks.	N=488 weeks.	N=292 weeks.
(M) Means.	Ratios. Discounts.	29.8115±0.1021 3.1120±0.0614	27.2869±0.0415 4.2234±0.0813	34.1130±0.1523 1.7038±0.0298
(σ) Standard Deviations.	Ratios. Discounts.	4.2285±0.0722 2.5429±0.0434	1.3891±0.0297 2.7288±0.0583	3.8566±0.1077 0.7556±0.0211
(r) Coefficients of Correlation.	Ratios and Discounts.	0.5231±0.0142	0.5920±0.0197	0.5997±0.0253
(R) Coefficients of Regression.	Discounts on Ratios.	0.3146±0.0124	1.1617±0.0572	0.1175±0.0062

§ 60. The regression lines are plotted in Diagram No. 15. In the case of the total number of weeks (N = 780), it is obvious that the regression line (DE) on the linear basis is not a good fit. The line of regression (slope R = 0.3146) is considerably higher in the middle and lower at the extremities than the actual records.

To remedy this, the total number of weeks was split into two parts. The major number of weeks (N = 488) fell below the mean of the ratio of reserves to deposits. The slope (R) of the regression line (F E) equals 1.1617. This line is a thoroughly satisfactory fit. The slope (G H) for the upper portion (N = 292, R = 0.1175) is slight, and the relation is comparatively of small importance.

Now theoretically the best fit for the whole system is a curve. But practically since the majority of the cases fall below the mean of the ratios of reserves to deposits 29.8115, and since it is in these cases that the regression is of real importance, simplicity of expression favors the straight line.*

* Pearson says in a somewhat similar study,—"Now the reader has only to look at our regression diagrams, in particular at that for brethren, to assure himself no curve will serve for practical purposes substantially better than the straight line. . . . . . All this is independent of any theory of frequency distribution, and the vanishing of (r) with the correlation simply flows from the fundamental problem that the chance of a combined event is the product of two independent probabili-

§61. What, in words, is the meaning of a coefficient of regression of 1.1617 in the case of call discounts on the ratio of reserves to deposits? It means simply that with every point of fall in the ratio of reserves to deposits call discounts tend to rise approximately one and one-sixth points. This relation is as authentic and as well justified by its appropriate facts and within its appropriate limits as the physical law of the expansion of mercury with the application of heat. It enables us to write the regression equation of call discounts on the ratio of reserves to deposits in a mathematical formula.

In the biological language* of Prof. Pearson,—"if we have a first organ of magnitude $m_1 = M_1 + x$, then the most probable value of the second organ (i. e. the mean value of its array) is $m_2 = M_2 + y$, where $y = xr\ \sigma_2/\sigma_1$.

"Thus the value of m_2 is given by $m_2 = M_2 + r\ \sigma_2/\sigma_1\ (m_1 - M_1)$. Such an equation is termed a regression equation, and R_{21} (i. e. $r\ \sigma_2/\sigma_1$) is termed the coefficient of regression. In words: the probable deviation of a second organ from its mean is the product of the coefficient of regression into the observed deviation of the first organ from its mean. When the regression is perfect, i. e. $m_2 = M_2$, the coefficient of regression, or the correlation r, must vanish. When the correlation is perfect, or $r = 1$, then the regression is least, or m_2 differs most from M_2."

Pearson's expression, $m_2 = M_2 + r\ \sigma_2/\sigma_1\ (m_1 - M_1)$, in the present case may be written

$$m_{\mathrm{d}} = M_{\mathrm{d}} + r\sigma_{\mathrm{d}}/\sigma_{\mathrm{r}}\,(m_{\mathrm{r}} - M_{\mathrm{r}}).$$

Substituting the values given for the letters in Table No. 25, we have

$$\begin{aligned} D &= 4.2234 - (1.1617)\,(R - 27.2896) \\ &= 35.927 - 1.1617R. \end{aligned}$$

Thus if we knew beforehand that the probable value of the

ties. . . . Our conclusions in this paper are deduced from the above value of (r) and from the slope of the regression line, and they involve no further assumption than the approximately linearity of the regression curves." (Roy. Soc. Proc., 1899, Vol, 65, 295.)

* *Grammar of Science*, p. 401.

ratio of reserves to deposits would be 25.0%, we could substitute the value R=25.0 and solve for (D), which in this case would be about 7%.

This equation simply states mathematically the rule, previously stated in words, that, whenever the ratio of reserves to deposits is below 29.5%, for every point of fall in the ratio, call discounts tend to rise approximately 1 and 1/6 points.

§ 62. A practical consideration of the above results may suggest several questions. Of what use, it may be said, beyond satisfying an academic curiosity, is this relation between the call discount rate and the ratio of reserves to deposits? It is an immediate correlation and not anticipatory. How, it may be said, not knowing the ratio, can we predict the discount rate?

The bearing of all this is seen at once when it is remembered that the ratio of reserves to deposits is but the quotient of these two quantities. The reserves form the numerator of the fraction, the deposits the denominator.

Consequently, knowing the average periodicity of the reserves, we can within limits predict the influence of the numerator on the ratio. The periodicity of the deposits has not yet been determined. But inasmuch as the deposits are, to a very large extent, the offset to the loans, and vary with the loans, knowing the periodicity of the loans enables us to study the influence of the loans upon the denominator of the ratio. Thus we may hope that, with a further development of the dynamic element, we may be able in time to predict the ratio within close limits, and ultimately the call discount rate.

As correlations among these items and other items in the equation of the societary circulation and in the foreign exchange field* are developed, the limits of error may be greatly reduced. Indeed, it requires only time and a sufficient number of investigations to reduce all this field of finance to an exact science.

§ 63. A consideration of the means and of the standard

* See § 70.

deviations in Table No. 25 brings out a striking antithesis. It will be noted that the high mean discount rate 4.2234 occurs with the low mean ratio 27.2869, and that the high mean ratio 34.1130 occurs with the low average discount 1.7038.

In the same manner, variability as measured by the standard deviation is subject to a similar antithesis. The small standard deviation of the ratios occurs with a high standard deviation for call discounts. This of course testifies to the urgent use the banks make of the discount rate as a weapon against falling reserves.

The coefficient of variation has been defined by Pearson to be the ratio of the standard deviation to the mean multiplied by 100. The following table shows how variability changes with the different classifications and also how distinctly the bond-rate is differentiated as a non-fluctuating type.

COEFFICIENTS OF VARIATION.

Bond-rates.	Weeks.	Call-rates.	Ratios R/D.
15	780	60	14
	292	44	11
	488	33	5

§ 64. As a business development of such statistical theory and investigation there would seem to be room for under-writing companies, whose chief business should be the selling of 'puts' and 'calls' on the discount rates for different periods of time. Such work, whether done by independent under-writing corporations or by banking houses, would be of very great avail in the flotation of new securities and in the operation of the pools which are so necessary to the existence of a great financial center.

Such a means of insuring the interest rate would do much to lessen the destructive effects of panics. For it would tend, like all insurance, to spread the losses and would do much to prevent the domino-series of failures, which attend on every such disturbance to credit.

CHAPTER VIII

CORRELATION BETWEEN THE CALL DISCOUNT RATE AND THE PERCENTAGE DEVIATIONS OF THE TOTAL RESERVES

§ 65. An *a priori* reason for correlation between the call discount rate and the reserve deviations flows from the part the reserves play in the numerator and, indirectly, in the denominator of the ratio of reserves to deposits.

Now, inasmuch as the correlation between the ratio of reserves to deposits and the discount rate was slight when the ratio stood above 30%, it seemed best to deal only with correlation between the discount rate and the reserve deviations when the ratio stood below 30%. The following table (No. 26) is the correlation table of the above items under the given condition.

TABLE No. 26.

Discount Rate.	Reserve Deviations.																
	−45	−40	−35	−30	−25	−20	−15	−10	−5	0	+5	+10	+15	+20	+25	+30	+35
1.0	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...
1.5	...	...	...	...	...	...	...	...	...	...	4	3	11	6	6	...	...
2.0	...	...	...	...	...	1	2	9	5	13	12	24	10	7	1	...	...
2.5	...	...	...	...	...	1	10	11	9	15	14	11	5	1	2	7	1
3.0	...	...	...	...	...	2	5	9	6	7	10	5	5	2	1	1	1
3.5	...	...	...	...	...	2	7	6	9	10	4	3	2	1	1	...	...
4.0	1	...	...	1	...	4	5	12	4	7	2	5	3	...	3	...	...
4.5	...	...	1	1	1	4	12	15	8	...	3	1	...	...	1	...	...
5.0	1	1	...	1	2	...	15	11	10	...	4	2	2	1	2	...	...
5.5	...	...	...	1	...	...	8	4	6	...	2	1	...	...	...	...	...
6.0	...	...	...	2	6	4	13	2	8	3	...	...	...	1	...	...	...
6.5	...	...	...	1	3	...	...	3	1	...	1	...	...	...	...	...	...
7.0	...	...	...	...	1	1	5	6	5	...	...	...	...	...	1	...	...
8.0	...	...	2	...	2	...	4	...	2	1	...	...	...	...	...	...	...
9.0	1	...	...	...	1	1	...	1	1	...	...	...	...	...	...	...	...
10.0	...	...	1	...	1	1	2	1	1	...	...	...	...	...	...	...	...
12.0	...	...	...	2	...	1	...	...	1	...	...	...	...	...	...	...	...
15.0	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
20.0	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...
25.0	...	...	...	...	2	...	...	...	2	...	...	...	...	...	...	...	...

§ 66. The regression averages of the call discount rate on the reserve deviations (Table No. 27) are drawn in Diagram No. 16.

TABLE No. 27.

Regression Averages of Call Discounts on Reserve Deviations.

Reserve Deviations.	Frequency.	Average Discount Rate.	Reserve Deviations.	Frequency.	Average Discount Rate.
−45	3	5.67	0	57	3.15
−40	1	5.00	+ 5	56	3.03
−35	4	7.00	+10	57	2.47
−30	9	7.50	+15	28	2.78
−25	19	8.34	+20	20	2.65
−20	22	5.18	+25	16	2.84
−15	99	4.44	+30	9	2.67
−10	91	4.14	+35	2	2.75
− 5	77	5.09			

The heavy line joining the solid dots is the regression polygon. The light line connecting the circled dots joins the averages, which can hardly be considered representative owing to their low frequency. The regression line is sensibly skew, and follows the general form of the regression polygon of the discount rate on the ratio of reserves to deposits. The two points previously remarked (§ 56) may be repeated.

(1) As the reserve falls, discount rates rise.

(2) The rate of increase is not uniform, but increases as reserves fall.

As in the previous case, a curve is the better fit, and, as soon as the method of moments is published, these averages will be fitted with an appropriate curve. Provisionally, however, the regression averages have been fitted on the linear basis. The following table (No. 28) contains the various constants of correlation.

TABLE No. 28.

	Reserve Deviations.	Call Discounts.
Mean, ...	-3.119 ± 3.986	4.123 ± 0.077
Standard Deviation, .	13.947 ± 2.818	2.691 ± 0.054
Coefficient of Correlation,	$r = -0.3707 \pm 0.0246$	
Coefficient of Regression,	Call Discounts on Reserve Deviations,	$R = -0.0715 \pm 0.0052$

Even on a linear basis the coefficient of regression amounts to 0.0715 with a probable error of 0.0052, or 7%. Diagram

No. 16 shows by the slope of the heavy line (AB) the coefficient of regression 0.0715.

This shows the basis underlying that close scrutiny of the movement of the total reserves by the operators and traders of the Stock Exchange. By experience they know that falling reserves, other things equal, tend to cause the discount rate to rise. In this light, the statement is favorable or unfavorable to advancing prices.

Inasmuch as the correlation is truly skew, it is hardly worth while to pay but a passing notice to the regression equation which follows:

$$D = 3.900 - 0.0715\, r.$$

D is the call discount rate for a given reserve deviation r.

To illustrate the form of the skew curve I have drawn (Diagram No. 16) the curve (M N) by freehand, which may stand for the regression curve on the skew basis. Now this curve (when it shall be expressed in a mathematical form) will perhaps be among the first supply curves derived from actual statistics. By supply curve, I mean the conception of Cournot. Estimates have been made of demand and supply curves from the times of Gregory King down to Marshall, but these have been simply estimates and for purposes of illustration. It is interesting to glance back across the space of over sixty years to the pithy sentences of Cournot.*

"Observations must therefore be depended on for furnishing the means of drawing up between proper limits a table of the corresponding values of D and P." (D = annual demand and P = average price); "after which, by the well known methods of interpolation or by graphic processes, an empiric formula or a curve can be made to represent the function in question; and the solution of problems can be pushed as far as numerical applications."

Here we have the supply† curve of interest, or better, the

* *Researches into the Mathematical Principles of the Theory of Wealth*, pp. 48.

† If the view is held that demand and supply curves are *statistically impossible*, then the above interpolation is a correlation, and nothing more. Indeed, Cournot's view, that demand curves for commodities may be derived from statis-

rate-price curve of the supply of call capital. Thus we may write $D=\phi(S)$. By the letter (D) we mean the discount rate and by the letter (S) the supply of capital. As soon as the mathematical formula of the curve is determined, we may substitute the exact, algebraic expression for the indefinite function $\phi(S)$.

By the above method the discount rate is quantitatively determined, on the supply side, by the static curve* of the supply of call capital on the New York Stock Exchange. The supply of call capital is, of course, measured in percentage deviations from the growth axis of the total reserves rather than in absolute figures.

In the same manner, the static curve of the demand for capital on the New York Stock Exchange, as represented by the percentage deviations of the loans, may be derived. The general form is represented in Diagram No. 17 by AB. The regression curve of discounts on reserves is represented by DC. The two curves are so drawn that the three means coincide, and the two ranges are extended to the same distances from the means on the figure. Now, if R is the reserve deviation at a time and L the loan deviation, the lines RF and LE represent the two rates of discount as determined by the two curves. We may, therefore, suppose that, so far as these two factors are concerned, the discount rate will lie between E′ and F. Thus we have two means of prediction, the second correcting the first. This is, of course, the case of three correlated variables. Corrections, however, are necessary on account of the fact that loan deviations are affected by changes in time as well as call loans.

This correlation between the reserve deviations and call discounts suggests an annual period in the call discounts to correspond with the period in the reserves. On this basis, the extremes of the discount period would seem to be about 1¾% and 5%, with the correlation, of course, inverse.

tics, is practically untenable. But the conclusion that it is impossible to derive demand and supply curves for capital at a price, whether call loans, railroad bonds or mortgages, by no means follows.

* Cf. Professor J. B. Clark's distinctions between static and dynamic conditions.

CHAPTER IX

CORRELATION BETWEEN THE RESERVE AND THE LOAN PERIODS

§ 67. So far we have been considering the correlation as immediate. A change in one item, we found, was accompanied by a change in the second item at the same time. We will now take up a case of anticipatory correlation. In Diagram No. 12, the periods of reserve and loan deviations are represented. This diagram suggests very strongly that the loan period follows the reserve period after an interval.

The differences between the two periods come out in this diagram. Since the total amplitude of the reserves is 19% of the growth axis or approximately 1/5 of the growth fund, the amplitude of the loan period is 4.7% or approximately 2/9 of the growth fund. Expressed as 19% and 4.7%, the latter is about 1/4 of the former and 4 : 1 makes a convenient scale for plotting.

The absolute scale in millions of dollars at the present time is easily found. The amplitudes are proportionate 4 : 1. The growth funds are proportionate 1 : 3. Consequently reducing the scale of the plot of the loans to 1 : 3, we should have a comparison in actual figures. For January 7, 1901, the million dollar amplitude of the reserves is roughly forty millions of dollars, and for the loans about thirty millions of dollars. It follows that the dynamic element of the loans is of far greater importance than the dynamic element of the reserves compared with their relative periodicities.

The tradition of probably higher money rates for the last weeks of December is also expressed in this chart. The rise in the loans during the last two weeks of December is much more rapid than in the reserves. The result is, the ratio of reserves to deposits is diminished, and high discount rates are the result.

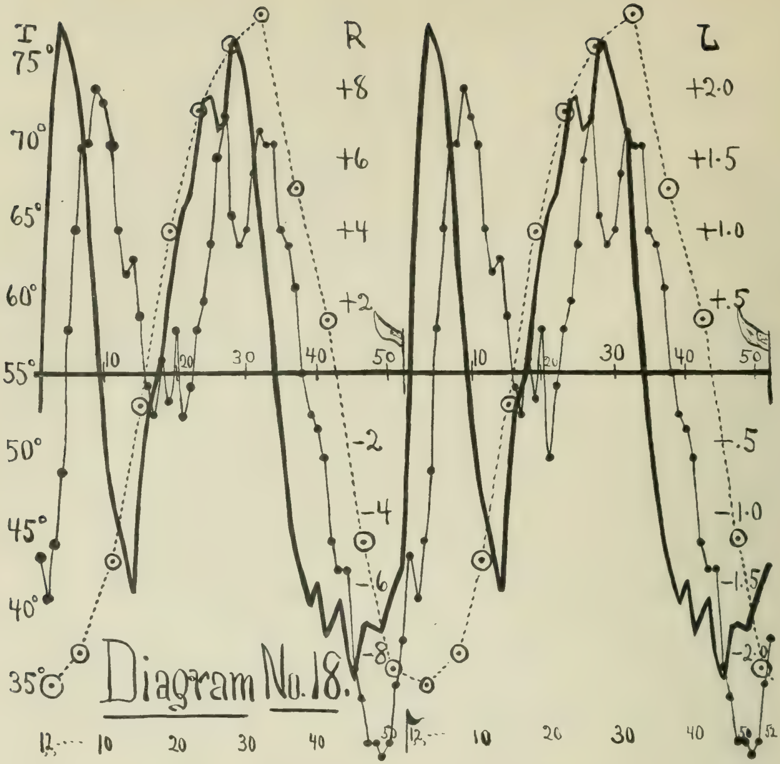


Diagram No. 18.

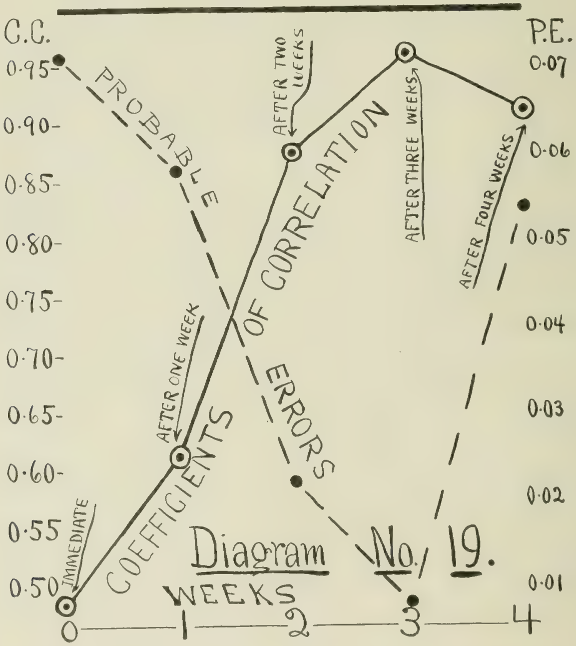


Diagram No. 19.

Roughly the loan period is the shadow of the reserve period with a three or four weeks interval. To show the wave motion which these two funds are undergoing with the seasons, I have drawn the periods for two years in succession. In Diagram No. 18, the heavy line represents the reserve period, the line broken by solid dots the loan period and the dotted line broken by circles the period of the temperature.

§ 68. If the loan period may be truly thought of as the shadow of the reserve period, it remains to discover how great is the interval. Does the loan index follow the reserve index of last week, two weeks ago, or a week still more remote?

The amount of the interval may be discovered by calculating the coefficients of correlation for lapses of one, two, three and four weeks. The correct interval is determined by the one having the highest coefficient of correlation. Table No. 29 contains the constants of correlation for the different intervals.

TABLE NO. 29.

Coefficients of Correlation (immediate and anticipatory) of Reserve and Loan Periods.

Correlation	after	Coefficients.
Immediate	———	$r=0.489\pm0.071$
Anticipatory	1 week.	$r=0.615\pm0.058$
"	2 weeks.	$r=0.872\pm0.022$
"	3 weeks.	$r=0.958\pm0.008$
"	4 weeks.	$r=0.914\pm0.054$

By Diagram No. 19, it is apparent that the highest correlation is reached after a lapse of three weeks, but that even to the fourth week there is a large degree of correlation. The existence of correlation after every interval is due to the continuous movement of the periods in one direction or the other for a considerable number of weeks. It is this that causes the almost perfect correlation (0.9575) after the three-week interval to flow over to the other cases.

A consideration of the form of the curve in Diagram No. 19 as well as of the changes that take place in the probable

errors of the several coefficients would seem to substantiate the statement, that the highest correlation between the two periods is reached after an interval of nineteen to twenty days.

This high correlation between the loan and the reserve periods throws a new light upon the meaning of the loan period. The loan period is not properly a period of the loans *sui generis*, but an ebb wave in the loans which follows after three weeks and is caused by the tide wave in the reserves. In other words, the loan period is really the shadow of the reserve period, and it is due, in a very large part, not primarily to the yearly period of time, but to a correlation (not yet worked out) which exists between the percentage deviations of the loans and the reserves. This correlation has apparently an interval of approximately three weeks.

This knowledge makes it clear that the genuine annual period of the loans should be derived by averaging the loan deviations, after corrections have been made for this correlation with the reserves. We should then have the loan material vibrating for reasons of its own, and not through the transmitted vibration of the reserves.

The reason for a correlation between the reserve and loan deviations is not hard to find in banking theory. Increasing reserves, we have seen, are correlated with decreasing call discounts. Now a falling discount rate is correlated with increasing loans. Consequently ensues the correlation between reserve and loan deviations after an interval of sufficient duration for the working out of these two correlations. Such an interval is common in physical phenomena. The regression equation of the indices of the loan period on the indices of the reserve period after a three weeks interval is

$$L_{t+3} = 0.2203\ R_t.$$

§ 69. If the reader will carefully follow out the movements of the lines in Diagram No. 18, a peculiar correlation between the temperature and reserve periods will be discovered. When the line of the temperature period is below the axis,

the line of the reserve period is in *inverse* correlation. When the temperature line is above the axis, the reserve line is in *direct* correlation. Nor is this species of correlation irrational. The sowing time comes before and about the point at which the line of the temperature crosses the axis on the rise, and the harvest time falls before and about the point at which the temperature line crosses the axis on the descent. In consequence, there is this *shuttle-cock* correlation.

The reserve fluctuation resembles a drop hammer. The drop hammer is released just before the blow is desired, and, when once the blow is struck, the hammer is raised to await the next release. We may think of the reserves as released at about the time of the turn of direction in the temperature. The blow is struck before the temperature has risen half its course, and the reserves rise to await the turn in the temperature. On the turn of the temperature for the descent, the reserves again fall. The blow is struck before the temperature has much more than crossed the axis, and the reserves again rise to await the next turn in the temperature.

§ 70. Among other correlations, which are believed to exist for reasons and upon which work* is being carried on are the following:—

Correlation between the reserve and loan deviations, immediate and anticipatory.

Correlation between the loan deviations and the call discount rate.

Correlation between loan deviations and the rate of foreign exchange (immediate and anticipatory).

Correlation between call discounts at New York and the rate of foreign exchange, as a case under the correlation of the call discount rate at London, Paris, Berlin and New York, and the rate of foreign exchange at each place.

* [If various statistical agencies would spend less time in working out annual averages and expend the economised clerk-hire in publishing tables of continuous daily or weekly figures, much labor would be saved the investigator by eliminating the toilsome task of collecting the statistics from the files of periodicals.]

Correlation between call discounts and sixty-day paper at New York.

Correlation between the rate of foreign exchange and the sixty-day paper rate at New York.

Correlation between the percentage deviations of the interior clearings and the reserve deviations, immediate and anticipatory.

§ 71. To compare the values of the coefficients of correlation, found in the last three chapters in our financial studies, with the values ascertained in several biological investigations, I submit Table No. 30, compiled from some of Pearson's Memoirs.*

TABLE NO. 30.

Comparison of Correlation Coefficients in Biology with those derived in the Preceding Pages.

(A) Pearson's Table of Correlation Coefficients in Man.	
Length and breadth of skull,	0.29 to 0.49
Breadth and height of skull,	0.10 to 0.34
Length and capacity of skull,	0.50 to 0.89
Weight and length (babies),	0.62 to 0.64
Weight and stature (adults),	0.50 to 0.72
Right and left femur bone,	0.96
Strength of pull and stature,	0.22 to 0.30
Strength of pull and weight,	0.34 to 0.54
Age at death of fathers and sons,	0.09 to 0.13
(B) Correlation Coefficients in the Preceding Pages.	
Ratio of reserves to deposits and discount rate (780 weeks),	0.52 ± 0.01
Ratio of reserves to deposits and discount rate (488 weeks),	0.59 ± 0.02
Ratios of reserves to deposits and discount rates (292 weeks),	0.60 ± 0.01
Reserve deviations and discount rate,	0.37 ± 0.02
Reserve and loan periods (immediate),	0.49 ± 0.07
Reserve and loan periods (after one week),	0.62 ± 0.06
Reserve and loan periods (after two weeks),	0.87 ± 0.02
Reserve and loan periods (after three weeks),	0.96 ± 0.01
Reserve and loan periods (after four weeks),	0.91 ± 0.05

It is apparent by the above table that the highest degree of correlation in the biological table, the correlation between

* Before the table of the biological coefficients of correlation (taken from *Grammar of Science*, p. 402) is the following statement: "I close this section with a table of a few coefficients of correlation in man, so that the reader may have some idea of the extent to which characters and organs in one type of life are correlated."

the right and left femur bones, is equalled by the correlation which we have found to exist between the reserve and loan periods.

Professor Pearson closes the chapters on evolution with these words.* "The reader who has followed the author through the somewhat difficult quantitative discussions of this and the previous chapters, will probably arise from the perusal with the conviction that biology is almost as *exact* as any branch of physical science."

If the biologist can point to these correlations as satisfactory scientific laws, it is hardly more than fair to grant the same privilege to the economist. In short, it seems reasonable to conclude that economics may become almost as *exact* as biology.

* *Grammar of Science*, p. 500.

CHAPTER X

THE CRISIS

§ 72. The dynamic elements, as already noted, cover the movements of the funds which do not come under the statistical laws of the growth axis or periodicity. These dynamical movements were classified as, case I, a cycle, i. e. an uncertain period of gradual change; case II, a catastrophe, i. e. a crisis with violent change; case III, minor dynamical changes.

Now it should be noted that although we have not been able to derive simple laws to express the dynamical movements correlate with continuous or recurrent time, nevertheless we should by no means despair of correlating these cases with movements in other items. But inasmuch as this field cannot be advantageously worked until the previous elements have been studied, it will be sufficient to point out at this stage a theoretical point of some practical interest.

A study of the disturbances in credit during the years 1884, 1890, 1893 and 1899 discloses a sequence of four phenomena: (i) a rapid fall in reserve deviations; (ii) culminating with high discount rates; (iii) a fall in the loan deviations; (iv) a readjustment of the above items with a rapid rise in reserve deviations to great proportions.

In the extent of these movements and in the level from which the movement starts, we have criteria for the definition of crises and panics and a means of comparing these disturbances in credit, as measured by banking barometers. Thus we may define a panic, in general, as a less violent fall in the reserves from a higher level of shorter duration, and a crisis as a more violent fall from a lower level of longer duration.

§ 73. As an illustration, I have prepared Diagram No. 20, showing the movement of the dynamic elements of the reserves during the years 1893 and 1899. The one year contains the crisis of 1893 and the latter year the panic of

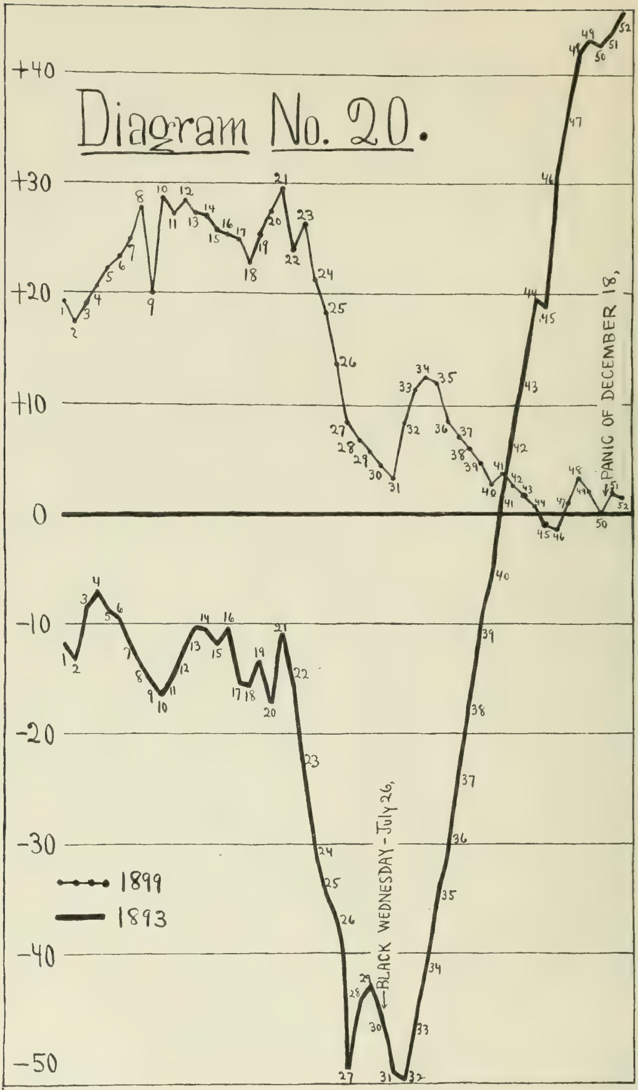


Diagram No. 20.
+40
+30
+20
+10
0
-10
-20
-30
-40
-50
1899
1893
BLACK WEDNESDAY - July 26,
PANIC OF DECEMBER 18,

December 18th. The dynamic indices are the differences between the percentage deviations and the periodic indices.

In comparing these two disturbances the two points already mentioned should be noted: (i) the level from which the disturbance starts, and (ii) the rate of decrease per week. The lower the level and the swifter the decrease, the more severe is the catastrophe. In the history of crises and panics, writers have called naturally sudden decreases from high levels panics, and violent and prolonged decreases from low levels crises.

The crisis of 1893 starts from the low level of approximately −15. The panic of December 18th, or the preliminary liquidation, started from a level of about +25.

Black Wednesday, July 26, 1893, occurred after a fall of forty points in six weeks, or an average fall of over six points per week. The panic of December 18th occurred after a fall of about twenty-five points, distributed over twenty-seven weeks, or an average weekly fall of a little less than one point per week. Diagram No. 21 (folding Chart) represents the reserve deviations for the crises of 1884–5 and 1893–4 on the same scale. The similarity is apparent. The sharp break causes discount rates to rise violently. Prices fall and failures ensue. The volume of business transactions diminishes and the currency which is no longer needed throughout the country to effect exchange accumulates at New York.

By these two criteria taken in connection with reserve and loan deviations and call and sixty-day paper rates, a very concrete measure may be made for determining the severity of such catastrophes.

A striking physical parallel is the barometer. In forecasting weather, the observer takes note of the level as well as of the sudden change. The barometric reading is a measurement of the atmospheric pressure, agreed upon by scientists. So the expansion of mercury is taken as a measure of heat. To compare heat by subjective feeling is the same as the comparison of two crises by men who have lived through both, and then state their opinions of the relative severities.

Here is a possible bank test—not perfect, for other causes enter in. But it is like the thermometer. It was 90° in the shade yesterday, is 80° to-day. The subjective experience, however, may profoundly differ on account of varying humidity, wind or other modifying influence. But as the thermometer is useful, so the bank test of crises may become of use. Even taking the reserve deviations alone, the level and the rate of fall show a decided difference between 1893 and 1899. 1893 represents the emaciation of adversity. 1899 stands for the "indigestion of prosperity."

CHAPTER XI

SUMMARY

In Part I an attempt was made to state clearly the leading factors of the societary circulation and to express as an approximation the relations of these factors in the equation of exchange-work. These factors may be thought of as parts of the delicate and complicated machinery which is constantly in operation in a modern society with the object of increasing the trade and diminishing the friction. The factors of the exchange-work equation, i. e. the frame-work of the machinery, are nearly all represented by letters in the equation, every letter having a certain numerical value for every moment of time. *A priori* analysis brings us as far as this equation of the relations of these factors. This serves as a systematic arrangement of the facts in preparation for the statistical analysis. The statistical analysis corresponds to the test tubes and mortars of the chemist, by use of which the component parts of a compound mixture are isolated and measured. At about this point, it is well to suspend theorizing and to commence collecting the facts. This is an onerous though necessary process; for each factor has a different value in each succeeding moment. The writer—as an illustration of some of the methods which should be applied to each and every letter—collected from the files of journals the statistics of several of the letters weekly for a series of years. These factors were the loans, deposits, reserves, ratios of reserves to deposits and call discount rates of the New York banks. These items were then subjected to statistical analysis to determine how each set of factors varied with the continuous lapse of time, with the recurrence of intervals of time, as the year, the quarter and the month, and with the movements (immediate and anticipatory) in other sets of statistics. As far as possible, these results have been graphically repre-

sented, but they have been in all cases numerically expressed and the probable errors have been generally attached.

Conclusions from the statistical analysis may be summarized briefly as follows:

(1). As a first approximation, the exchange-work equation affords a starting point for this as well as future investigations. In the present stage of economic science, such investigations of the facts will, it seems to the writer, be far more fruitful than further eclectic research of authorities. The motto of the *Biometrika* may well be taken over bodily into economics: "*Nihil est quod non numerandum.*"

(2). Indices for measuring the successive credit strains to which the national banking system of the United States is subjected, are one of the fruits of the equation of exchange-work. These percentages are believed to bear important relations to the movements of prices, interest rates, the foreign exchanges, and the import and export of specie.

(3). The use of the growth axis to represent solely the element of growth, rather than all of the statistical movements, is believed to have many applications to all kinds of continuous statistics, both economic and social, that are subject to rates of increase. By thus eliminating the influence of growth, the percentage deviations can be prepared for the application of Pearson's correlation methods, which have produced so many strikingly original results in evolution.

(4). In the interpolation of the growth axis, a qualitative test of fitness seemed on the whole truer to the representation of the curve as a scaffolding for the statistics, than the commonly accepted test of least squares.

(5). The percentage deviations represent those fluctuations which affect human interests most vitally. They are more evident; thus the reserve deviations travel up and down on an average of twenty points while the growth axis is advancing one point. The removal of the growth element reduces the deviations to an undulatory type.

(6). The annual period of the reserve deviations, long known to bankers by experience but hitherto unmeasured,

is clearly revealed. The normal, annual period consists of a double fluctuation in which the distance between the extremes is approximately 18% of the growth axis; thus, during a year, the reserve deviations travel up and down the yearly period a distance which is between 60 and 70% of the growth axis.

(7). An annual period in the loans, corresponding to and believed to be the result* of the period in the reserves, has been isolated and measured. The loan period is, however, relatively less important than the period of the reserves; thus, in the course of a year, the loan deviations travel up and down the normal period a distance which is between 15 and 20% of the growth axis.

(8). The statistical analysis has also suggested quarterly and monthly periods in the loans, but the indices are believed to be suggestive rather than quantitatively representative.

(9). The specialization of loans into various types, with the result that all the funds of a people are constantly productive, is illustrated by the contrast between call and bond loans both with respect to duration, actual rate of interest and variability of rate.

(10). The importance of the ratio of reserves to deposits as a criterion of the money-market, long recognized by the operators, is numerically demonstrated. During fifteen years, the coefficient of correlation has stood above 0.50 with a probable error of 0.01. Further, call discounts tend to rise much more rapidly as the ratio decreases absolutely.

(11). The importance of the bank statement is numerically emphasized by the high correlation between the reserve deviations and the call discount rate. It is believed that there is, further, an anticipatory correlation of some importance between reserve deviations and call rates.

(12). The importance to the public of the weekly bank statements, as a barometer of credit conditions, is quite

* As far as possible, mention of cause and effect has hitherto been excluded; for the primitive fallacy, *post hoc ergo propter hoc*, is nowhere more dangerous than in work of this nature.

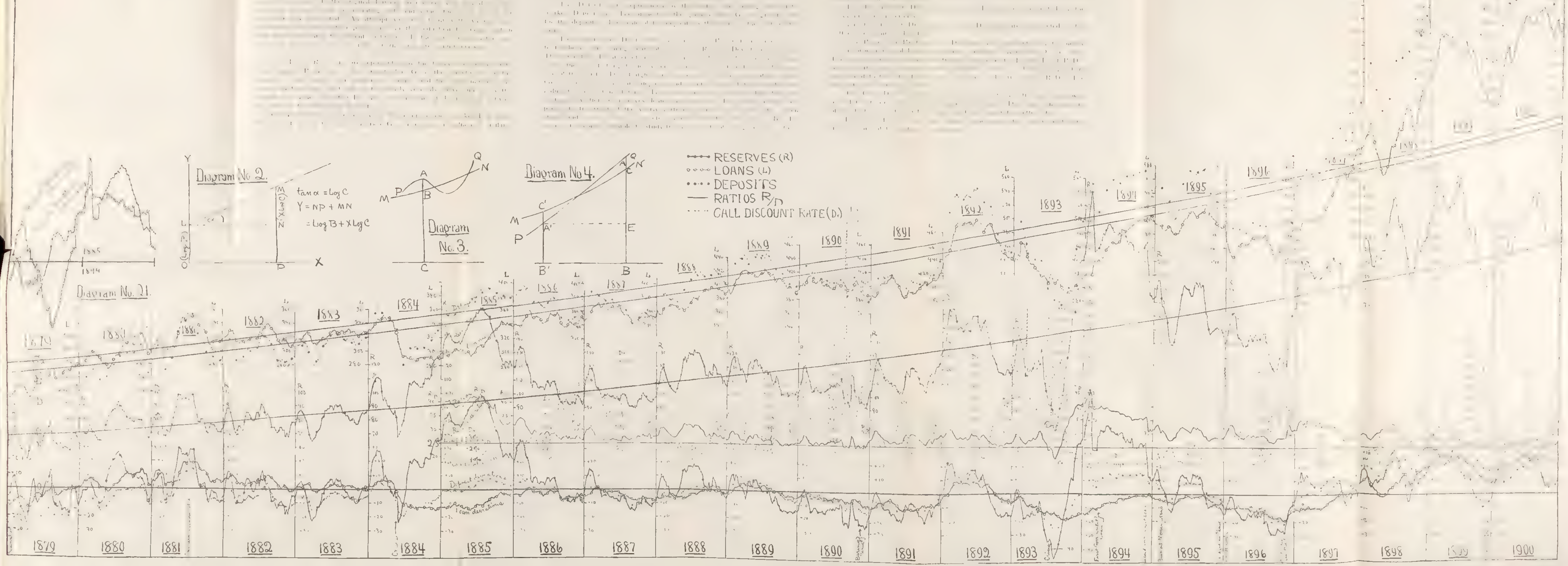
obvious; because the recent rapid growth of trust companies brings about the possibility of the concealment of loans and thus causes the bank statements to be no longer representative for this item in the money-market, force is lent to the movement for weekly statements by the trust companies.

(13). The almost perfect correlation between the reserve period and the loan period is interpreted as cause and effect. The *shuttle-cock* correlation between the reserve period and the temperature period emphasizes the importance of climate even on occupations so far removed from the soil as banking. It is supposable that this banking period would undergo considerable modification for a climate where the crops were raised continuously.

(14). The peculiar likeness of panic to panic and crisis to crisis, even during the few years covered by this study, suggests a field for very careful gathering of facts on an extensive scale. The crisis is so fraught with vicissitudes to men that it seems worth while to seek knowledge from the past in the hope of finding adequate safeguards.

(15). The high correlations among economic phenomena, immediate and anticipatory,—a few instances are presented in the preceding pages and many more are in process of verification but as yet unpublished—suggest strongly that foresight in business may in the future be vastly increased. Indeed, the phenomena of economics lend themselves far more readily to the possibility of prediction than do the phenomena of meteorology. Such prediction, however, can come only through the finer methods of statistical analysis and through concerted action in the assembling of the facts.

Chart Showing the Movements of the Weekly Averages of the Total Reserves, Loans and Deposits of the New York Associated Banks (1879-1900), with their Respective Growth Axes; also Percentage Deviations of the Total Reserves, Loans and Deposits (1879-1900), Ratios of Reserve to Deposits (1885-1900), and the Discount Rate on Call Loans at the Stock Exchange (1885-1900).



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